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NOTE ON PROBABILISTIC ARGUMENTS FOR THE EXISTENCE OF GOD

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A b s t r a c t. The article presents the theistic arguments for the existence of God that have been developed for the past 400 years and that use the concepts from probability theory. It focuses on: 1) the probabilistic version of the argument from design, and 2) Pascal's Wager. In order to present these two arguments rigorously, the so called probabilistic measures of rationality have been defined. The measures are based on the concepts of (classic) probability, conditional probability, and the expected value. These concepts have been applied to reconstruct the arguments in question. Additionally, the analytic form of the formulas defining rationality measures enables to discuss the substantive value of the arguments presented. Their weight of evidence depends to a great extent on the ontological, epistemological, and cultural assumptions that have been accepted either explicitly or implicitly. Pascal's Wager seems to be the most resistant to criticism. It also appeals to a wide range of people. Still, it is largely based on the accepted world view.

Key words: probability, Pascal's Wager, argument from design.

Natural theology has always searched for rational arguments to substantiate its claims. It is, therefore, not surprising that since the concept of probability

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was formulated in mathematics and then used in the empirical sciences, it has also been embraced in theological argumentation.

The article attempts to reconstruct some theistic arguments for the existence of God applying the means and methods of modern probability theory. The concept of probability was invoked in those arguments either explicitly or implicitly, but it lacked rigour. That is why, it seems necessary and desirable to explain and, as far as possible, formally present those reasonings.

1. THEORIES OF THE CONCEPT OF PROBABILITY

Blaise Pascal (1623-1662), Pierre de Fermat (1601 or 1607-1665), and Christian Huygens (1629-1695) were the founders of the classical probability theory. They began systematic studies in that area in the mid 17th century[1]. The first work to deal with the new branch of mathematics was a book written by Huygens, *De ratiociinis in ludo alea* (1657). However, it was Jacob Bernoulli's (1654-1705) *Ars conjectandi* published posthumously in 1713 that is considered to be a milestone in the development of probability theory. This work contained basically all the essential concepts of the classical probability theory, anticipating the so-called classical definition of *a priori* probability, which was later on presented in detail by Pierre Simon de Laplace (1749-1827) in his *Théorie analytique des probabilités* (1812). In 1763, an article entitled *An Essay towards Solving the Problems in the Doctrine of Chances* by Thomas Bayes was published. It contained a famous formula expressing the so-called *a posteriori* probability (this formula was given in the form slightly different from the one that we know today). In this way, grounds for the so-called classical probability theory were formed, a theory that served as the basis for the axiomatic theory of probability that developed in the 20th century mainly thanks to Andriei Kolmogorov (1903-1987).

How did the founders of the classical theory of probability view the concept of probability itself? Jacob Bernoulli considered this subject in the third part of *Ars conjectandi*, supporting the view of strict determinism of natural

[1] L. GRUSZECKI. *Zarys dziejów matematyki. Logika. Teoria mnogości. Liczby i numeracja. Algebra. Geometria. Rachunek różniczkowy i całkowy funkcji zmiennej rzeczywistej. Funkcje zespolone. Prawdopodobieństwo*. Lublin: Wydawnictwo KUL 2012 p. 289-328.

phenomena. According to him, all events of the past, future, and present are a necessary work of Divine Providence. Thus, the *a priori* probability is only the measure of our inter-subjective ignorance about the phenomena and laws governing them. Similar views on the nature of probability were expressed by Laplace, who, however, did not share Bernoulli's religious beliefs. The concept of probability in such form was used either explicitly or implicitly by, among others, Blaise Pascal, authors of the *Boyle Lectures* or the *Bridgewater Treatises on the Power, Wisdom and Goodness of God as Manifest in the Creation* as well as by William Paley (1743-1805) and David Hume (1711-1776)[2] in their theological and philosophical arguments for or against the existence of God. In the 20th century, similar arguments were also employed by F.R. Tennant (1866-1957)[3], Pierre Lecomte du Noüy[4], and Richard Taylor[5]. They all supported the so-called *argument from design*, an argument that has been invoked more recently by Francis S. Collins[6], Paul Davies[7], or Arthur Peackock[8]. In recent decades, the concept of probability has come to be understood not only as the measure of our ignorance about the world, but as the internal property of matter that reveals itself primarily at a quantum level.

It should also be noted that the 20th century brought some attempts to develop different approaches to the concept of probability formulated as if on the peripheries of the axiomatic theory of probability. John Maynard Keynes (1883-1946), Rudolf Carnap (1891-1970), and Jaako Hintikka (b. 1929), to name a few, tried to develop the theory of probability on the grounds of logical and linguistic considerations; theories of that kind examined the con-

[2] *The Boyle Lectures* were a series of lectures endowed by Robert Boyle (1627-1691). *The Brigewater Treatises* were published between 1833 and 1840 and consisted of 8 volumes. Paley was the author of *Natural Theology, or Evidences of the Existence and Attributes of the Deity, collected from the Appearances of Nature*, among others, which was published in 1802. Hume's views on this issue were presented in *The Dialogues Concerning Natural Religion*. Cf. also J. HICK. *Arguments for the Existence of God*. London: Macmillan 1970 Ch. 1 and 2.

[3] F.R. TENNANT. *Philosophical Theology*. Vol. II. Cambridge: Cambridge University Press 1930.

[4] P. LECOMTE DU NOÜY. *Human Destiny*. London–New York: Longmans 1947.

[5] R. TAYLOR. *Metaphysics*. London–New York: Prentice Hall 1963.

[6] F.S. COLLINS. *The Language of God*. New York–London–Toronto–Sydney: Free Press 2006.

[7] P. DAVIES. *God and New Physics*. New York: Simon & Schuster 1983.

[8] A. PEACOCKE. *Paths from Science Toward God. The End of All Our Exploring*. Oxford: Oneworld Publications 2001.

ditional probability of certain states of things that were described by some statements, given that the other statements about them were true. On the other hand, Richard von Mises (1883-1953) based the concept of probability on the concept of event frequency, an approach that is sometimes traced back to Aristotle (384-322 BC). According to this approach, the probability of a random event is the limit of a sequence of empirically specified frequencies of observed variants of that event. The concept was later alluded to by Hans Reichenbach (1891-1953), among others. The frequentist theory met with a lot of resistance, as it did not clearly separate the empirical content from formal concepts, such as the limit of a sequence. Still another view on the fundamentals of probability was proposed by Bruno de Finetti (1906-1985). He started a debate on the so-called subjective approach to the concept of probability. Bruno de Finetti expressed his views primarily in the book *Probability, Induction and Statistics: The Art of Guessing* (1972). According to him, a probability does not exist objectively as the property of reality, neither is it an inter-subjective idea shared by all in the same way. Every person has their own assessment of a probability, which may be different from the assessments of other people. However, this subjectivity also has its limitations, because it is possible to transform probabilistic beliefs of one person into beliefs of another one by means of conditional probabilities.

The concepts of probability mentioned above are not very effective in mathematical terms since they can be applied only in elementary cases. This fact must also affect their evaluation in philosophy[9]. Hence, we will further refer to the concept of probability that has been developed within mathematics and has been effectively used in the empirical sciences such as physics and chemistry, and social sciences such as sociology or economics.

2. PROBABILISTIC MEASURES OF RATIONALITY

Here for simplicity, we will rely on the classical definition of probability formulated by Bernoulli and Laplace. According to the definition, a proba-

[9] R. WEATHERFORD. *Philosophical Foundations of Probability Theory*, London: Routledge and Keagan Paul 1982.

bility space Ω consists of finitely many, say n, elementary events w_1, w_2, ..., w_n, i.e.

$$\Omega = \{w_1, w_2, ..., w_n\}$$

where individual elementary events are all possible indivisible and equally probable states of things. Thus, if A is an arbitrary (not necessarily elementary) event, then its probability is given by

$$P(A) = \frac{K_A}{n},$$

where K_A is the number of elementary events making up A.

This definition allows us to formulate *the first,* most elementary, *criterion for rationality* based on the concept of probability. If H is used to denote some hypothesis about the state of things, then interpreting this hypothesis as referring to one of the possible events, we assume that $H \subset \Omega$ and

$$P(H) = \frac{K_H}{n}$$

is the (probabilistic) measure of H's rationality.

Thus, if $P(H)$ is close to 1, or at least greater than 1/2, then insisting on the judgement corresponding to that hypothesis should be considered rational. Furthermore, out of two hypotheses H_1 and H_2 about the same aspect of reality, we should choose the one for which the probability is higher.

Such probabilities are called *a priori* probabilities because determining them is not based on any prior assumptions about the reality.

In fact, however, we assess hypothesis H on the basis of our knowledge about the world or on the basis of assumptions that seem to be obvious to us. These presuppositions will be further on denoted by S. So, we make use of the conditional probability, or *a posteriori* probability, this being *the second criterion for rationality*[10].

Therefore, out of the two hypotheses H_1 and H_2, the one for which the *a posteriori* probability is higher seems to be more rational.

When we speak of religious beliefs, it is not only important to express a particular belief, but also to live our lives according to this belief. As a ru-

[10] Of course, we assume that $P(s) > 0$.

le, an individual believing in the transcendent God acts differently than an atheist, who makes his life choices following different standards. The concept of a random variable as a function assigning the values of some ordered set, say a set of real numbers R, to elementary events, can be used to model the rationality of actions (this, in fact, was done by Pascal in his famous Wager, which will be discussed later). Accordingly, a random variable X is a function such that

$$X: \Omega \to \mathbb{R}.$$

The measure of rationality of selecting X is the so called expected value of this variable, i.e. the number

$$EX = x_1P(w_1) + x_2P(w_2) + \dots + x_nP(w_n),$$

where $x_1, x_2, \dots, x_n$ are assessments of decision values corresponding to elementary events or, in other words, elementary states of things. This constitutes *the third measure of rationality* constructed on the basis of *a priori* probability. The third measure can be further modified by using *a posteriori* probabilities, giving *the fourth rationality measure* in the following form

$$E_sX = x_2P_s(w_1) + x_2P_s(w_2) + \dots + x_nP_s(w_n).$$

3. THEISTIC ARGUMENTS FOR THE EXISTENCE OF GOD THAT USE PROBABILISTIC MEASURES OF RATIONALITY

We will use the measures of rationality defined on the basis of *a posteriori* probabilities, as they are more accurate and adequate in reflecting the nature of argumentations presented below. The analysis of two arguments of particular importance will be presented: 1) the probabilistic version of the argument from design, and 2) Pascal's Wager.

Let us start with the first argument, which can be summarized in the following way: the world which surrounds us and which we are a part of, is ordered, harmonious and beautiful; therefore, it is improbable that it came into being by itself. Consequently, it must have been created by some transcendent Intelligence that we call God. This argument appeared in many versions, with a probabilistic element not always emphasized. Some versions of the argument show the connection between order and harmony of the world and the existence of the Creator as not only very likely, but even logically compelling, for example the famous Paley's argument, in which he used an analogy between the Universe and a watch found on a moor[11].

Direct references to the concept of probability can be seen in the works of Tennant and Lecomte du Noüy. The latter calculates the probability (very small) of the appearance of a protein molecule, in a combinatorial manner. In both cases, it is demonstrated that the probability

$P_s(H)$

is very high, where:

H = God exists,
$S = S_1$ and S_2.

The first assumption expresses the assertion concerning the structure of the universe, i.e.

S_1 = the universe is ordered and harmonious, while S_2 expresses some theistic beliefs that a transition can be made from the intelligent mind designing the world to the good God the Creator. In other words, it assumes that a demon deceiver, for example, or numerous gods cannot be designers of the world, or that the intelligent mind is immanent in the world. Thus,

S_2 = if the objective reason lies at the foundations of the universe, then this objective reason is the good God the Creator.

For many people, this last assumption has convincing justification in religious practices of the majority of society, and in particular in the message of Christian revelation.

Let us consider the legitimacy of the very definition of probability, without calculating its value at this time. After all, as it was already noticed by Hume, creating of the universe cannot be compared to a dice throwing because we know only one universe, and the concept of a single dice roll seems

[11] W. Paley. *Natural Theology* (1802), abridged ed. by F. Ferré, Indianapolis: Bobbs-Merrill 1963, Ch. 1.

to be contradictory in itself. Speculations about the existence of some multiverse composed of a multitude of universes not communicating with each other, seem to be a futile entertainment of imagination. If the universe is one and unique, then

$$P_s(H) = 1 \text{ or } P_s(H) = 0,$$

and such equalities can be confirmed only empirically. In what way? We must content ourselves with the knowledge that one of the equalities is true, but we are not able to say which one.

However, Alvin Plantinga (b. 1932) noted that the fact that the universe was unique, did not mean that it did not share certain properties with other objects[12]; for example, let us add, with Paley's watch. Perhaps, it is possible to build a probability space that would contain not only one object, namely the universe where we live, but many objects? Logically speaking, it seems to be possible. How can we calculate the probability of it? Not to go into technical details, this task seems to be hopelessly difficult; especially because the tendency of the matter to organize itself can be observed in animate and inanimate systems. How, then, can we distinguish systems organized by some external factor from those organised by themselves?

We will now attempt to reconstruct what is possibly the most important of probabilistic arguments, namely Pascal's Wager. It was presented by the eminent philosopher and mathematician in his *Thoughts*, published posthumously[13]. In Chevalier's edition, the argument is included in the section titled *Infinite – Nothing. Wager* ([451]). Pascal compares man's life and religious and ethical choices that man has to make to a game of chance. He uses words such as gain, loss, or chance, terms familiar to him because his interest in probability theory resulted from his study on games of chance, and in particular on how to divide the stake among the players when the game is interrupted, and on the so-called Chevalier de Méré's problem[14]. Those considerations of Pascal implicitly gave rise to the concept of expected value, a concept formulated more precisely by Huygens in 1657, and later by Jacob

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[12] A. PLANTINGA. *God and Other Minds*. Ithaca, N.Y.: Cornell University Press 1967 p. 101.

[13] B. PASCAL. *Oeuvres complétes, texte établi et annoté par Jacques Chevalier*. Paris: Bibiothéque de la Pléiade, Libraire Gallimard 1954 p. 1081-1358.

[14] L. GRUSZECKI. *Zarys dziejów matematyki* p. 295. Gombaud Antoine Chevalier de Méré (approx. 1610-1648) was a philosopher, writer and a gambler.

Bernoulli in 1713. It is the concept of expected value that was implicitly used by Pascal in his Wager.

Let us recall what the Wager is about: God either exists or does not exist. If God exists, then we should live our lives according to strict rules and ethical and religious requirements following from faith. This, in turn, entails renouncing some (perhaps even many) worldly goods, but instead we gain immeasurably more: eternal life with God. On the other hand, if God does not exist and we erroneously choose Him, we lose on many earthly joys, but these joys are nothing compared to a possible, though not realized, reward. Therefore, we should always choose a virtuous life according to the precepts of faith. Such a choice is rational and follows directly from what was referred to earlier as the *fourth principle of rationality* based on the concept of probability.

Let us reconstruct Pascal's reasoning and apply to it modern mathematical symbols[15]. Firstly, let us note that his Wager is preceded by a discussion on the concept of infinity. Pascal states that infinity transcends everything and it cannot be increased by any value added to it. Today, referring to the foundations of mathematical analysis and using John Wallis' (1616-1703) symbol for infinity, this last relation can be written as

$$\infty + a = \infty ,$$

where a is an arbitrary finite number. Moreover, it is agreed that

$$a \cdot \infty = \infty ,$$

if only $a > 0$, and

$$0 \cdot \infty = 0$$ [16].

Let then H denote the hypothesis that God exists and H' the opposite hypothesis, i.e. that God does not exist. As previously, the symbol X stands for a choice made by man. This time, however, let us assume that

$$X\colon \Omega \to \mathbb{R} \cup (\infty).$$

[15] L. GRUSZECKI. *Nota o Zakładzie Pascala*. „Studia Philosophiae Christianae" 1998 Vol. 34. No 2 p. 171-174.

[16] The last equality is of fundamental importance in the contemporary theory of measure and integration, in particular Lebesque's measure and integration.

Wallis' symbol represents the infinite goods that are bestowed on the redeemed in Heaven, whereas $L > 0$ denotes the benefits gained when choosing the things of this world.

What is then the very essence of Pascal's Wager, i.e. what is the calculation of possible infinite rewards and possible finite losses? First, let us examine the presuppositions of this reasoning. Pascal does not explain the assumptions of S_1, although he probably does not ignore them either. However, the assumptions of S_2 are supplemented with additional beliefs, namely: 1) a virtuous life results in eternal reward while life dedicated to earthly pleasures deprives man of this reward, 2) Pascal, when writing about choices made by man in the context of the Wager, considers these choices to be completely free and not „forced" by the irresistible Grace. This last assumption seems to be even more important in view of the fact that Pascal declared himself to be a supporter of arguments proposed by Jansenists with their emphasis on double predestination.

With the above reservations, Pascal's calculations seem to have the following form

$$E_sX = P_s(H) \cdot \infty + P_s(H') \cdot (-L).$$

If we assume that $P_s(H) > 0$, and who, excluding the professed atheists, will not do that, then

$$E_sX = \infty .$$

If, assuming that $P_s(H) > 0$, we ignore God and opt for sinful pleasures, this choice – denoted here as Y – leads to the following result

$$E_sY = P_s(H) \cdot 0 + P_s\,(H') \cdot L = P_s(H') \cdot L < \infty .$$

Obviously, the strategy X, which gives much greater rewards, is more rational. The above calculations leave out the possibility of eternal damnation; in that case, the account would be even worse.

The above reasoning is based on an analogy. It is practically impossible to construct a suitable probability space that would contain hypotheses H and H', and to assign strictly defined probabilities to them. But the cogency of Pascal's argument lies precisely in the fact that these probabilities do not need to be clearly defined. It is enough to assume that $P_s(H) > 0$, i.e. even very small faith. Such an existential situation is typical for most people, apart from clearly professed atheists, for whom $P_S(H) = 0$, (in the case of professed atheists, presuppositions S will take a different form, anyway). For atheists,

$$E_S X = P_S(H') \cdot (-L) \text{ and } E_S Y = P_S(H') \cdot L ,$$

then

$$E_S Y > E_S X ,$$

so renouncing sinful worldly goods because of the belief in God is not profitable.

At this point it should be noted that an individual may have some motives other than religious faith and the prospect of eternal reward, to live a virtuous life. This issue, however, is not dealt with here.

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NOTA O PROBABILISTYCZNYCH ARGUMENTACH NA RZECZ ISTNIENIA BOGA

Streszczenie

Artykuł przedstawia, rozwijaną na przestrzeni ostatnich czterech stuleci, argumentację teistyczną na rzecz istnienia Boga, wykorzystującą pojęcia o charakterze probabilistycznym. Na szczególną uwagę w tym kontekście zasługują: 1) probabilistyczna wersja argumentu z celowego zamysłu oraz 2) Zakład Pascala. W celu precyzyjnego zaprezentowania obu dróg argumentacyjnych zdefiniowane zostały w artykule tzw. probabilistyczne miary racjonalności. Opierają się one na pojęciu prawdopodobieństwa (klasycznego), prawdopodobieństwa warunkowego i wartości oczekiwanej. Przy zastosowaniu tychże pojęć zrekonstruowane zostały wspomniane wyżej rozumowania. Analityczna postać formuł określających miary racjonalności umożliwia także przeprowadzenie dyskusji merytorycznej wartości omawianych argumentów. Ich siła dowodowa jest w znacznym stopniu uzależniona od przyjmowanych *explicite* i *implicite* założeń ontologicznych, epistemologicznych oraz kulturowych. Najbardziej odpornym na krytykę wydaje się Zakład Pascala, który przemawia do szerokiego spektrum ludzi. Jednakże również ten argument jest w istotnym stopniu oparty na założeniach światopoglądowych.

Słowa kluczowe: prawdopodobieństwo, Zakład Pascala, argument z celowego zamysłu.