ROCZNIKI FILOZOFICZNE
Tom LXVI, numer 3 – 2018
DOI: http://dx.doi.org/10.18290/rf.2018.66.3-4

MAREK LECHNIAK

ONCE MORE ABOUT MOORE'S PARADOX IN EPISTEMIC LOGIC AND BELIEF CHANGE THEORY

Moore's problem (paradox)[1] is examined on two levels: epistemology and logic. What is interesting is that the authors of the particular approaches practically do not take into consideration any alternative option. In the first type of approaches scholars attempt to explain the origins and reasons of the paradox as well as analyze their consequences for the theory of knowledge and beliefs (e.g. Moore's paradox is recognized as a serious argument against the representationalist concept of mind) (SCHMIDT 2014).[2] In its formulations in the field of logic (formal theory of belief change) the goal of the inquiries is different; it is the examination of minimal constraints that a formal belief model must fulfill such in order for it not to be possible for Moore's sentence to be proven as well as what are the logical reasons for its occurrence.

In this article I shall first present Moore's paradox *per se* and after that I shall concentrate on the logical perspective — at first I shall analyze these considerations in the field of so-called standard epistemic logics and after that I shall concentrate on the formal theory of belief change.

Dr hab. MAREK LECHNIAK, prof. KUL — Katedra Logiki na Wydziale Filozofii KUL; adres do korespondencji: Al. Racławickie 14, 20-950 Lublin; e-mail: lechmar@kul.pl

[1] Id est: paradox of belief as opposed to so called paradox of analysis (also called the Moore's paradox). Cf. LANGFORD 1952, 321–42.

[2] According to Schmidt "Although these views [various representations of the source of the paradox presented by this theory — M.L.] radically diverge in detail, they nonetheless arrive at the same explanation of Moore's Paradox. This explanation, however, is false. I will argue that any approach to Moore's Paradox committed to the representationalist view of mind and its inherent referentialist view about meaning must arrive at this explanation and that this provides sufficient reason to dispense with the representationalist theory of mind." (SCHMIDT 2014, 37).

1. FORMULATIONS OF MOORE'S PARADOX

As for the historical side of the issue, one might say after Thomas Baldwin (1993, 211–12) that the first mentions on the topic of the 'paradox' appeared during Moore's lectures in the 1930's. Moore explicitly formulated the paradox in 1942 in a reply to his critics in the volume *The philosophy of G.E. Moore* (SCHILPP 1952, 542–43) as well as (as it later turned out) in another version in the text from 1944 *Russell's Theory of Descriptions.*[3] The most complete considerations on the paradox and its origins may be found below, in the fragments of Moore's unfinished manuscript (stored in the Cambridge University Library) coming from, according to Baldwin, 1944.[4]

Moore's paradox itself can be presented in the following way:

> […] it's perfectly absurd or nonsensical to say such things as "I don't believe it's raining, but as a matter of fact it is" or (what comes to the same thing) "Though I don't believe it's raining, yet as a matter of fact it really is raining." I'm just assuming that it is absurd or nonsensical to say such things. But I want it noted that there is nothing nonsensical about merely saying these words. I've just said them; but I've not said anything nonsensical. And W. pointed out another proof that there isn't. He pointed out (I think) that there's nothing nonsensical in saying "It's quite possible that though I don't believe it's raining, yet as a matter of fact it really is" or "If I don't believe it's raining, but as a matter of fact it really is, then I am mistaken in my belief." In all these cases the very same identical words are said, but they are said in a context with other words, so that there is nothing nonsensical about them. (BALDWIN 1993, 207).

As one can see, Moore emphasizes that the statement "I don't believe it's raining, but as a matter of fact it is" is absurd, but it is not nonsensical (in a syntactic sense) each of its parts has a meaning and the entire expression is constructed properly according to the rules of syntax; its absurdity, apparent at first sight, requires, nevertheless, reconstruction and the reasons for it

[3] In SCHILPP 1944, 175–76.

[4] Baldwin writes as follows: "There is a letter from Wittgenstein to Moore written in 1944 in which Wittgenstein compliments Moore for a paper he had just given to the Moral Sciences Club about this matter. Could this be the manuscript of this paper? I doubt it—since it presents itself as a response to some remarks of Wittgenstein on this subject. It seems to me more likely that, following Moore's first paper (which is still missing), Wittgenstein gave a paper on the same subject; and then Moore came back to the matter with this paper. Wittgenstein discusses 'Moore's paradox' (the name comes from Wittgenstein) in section x of part II of his *Philosophical Investigations* (Blackwell, Oxford: 1953), which is his selection from the manuscript published as his *Last Writings on the Philosophy of Psychology*, vol. 1 (Blackwell, Oxford: 1982), where there is more discussion of Moore's paradox." (BALDWIN 1993, 212).

should be explained. Of course the sentence in quotation marks is not absurd[5]; only the utterance of such a statement is absurd. According to Moore the source of absurdity is the 'pragmatics of assertion,' i.e. the fact that somebody states something implies that the stating subject is convinced that what he or she states is true. "The words 'I don't believe it's raining' when said by a particular person have a definite meaning in English: we can say that what they mean is something about his state of mind — what they mean can't be true unless his state of mind is one which can be properly described by saying he doesn't believe that; and so with 'as a matter of fact it is raining.' The meanings of the two sentences are such that both can be true at the same time." (BALDWIN 1993, 209). This explanation may be abbreviated to the form suggested by Williams "In saying p one normally suggests that one believes that p, or expresses a belief that p." (WILLIAMS 1979), 141–42).[6]

According to Moore the reason for the paradox lies in the principle having an empirical character, "viz. that in the immense majority of cases in which a person says a thing assertively, he does believe the proposition which his words express." (BALDWIN 1993, 210). However, as John Searle indicates: linking the action of asserting something with the state of being convinced is not only the issue of a larger quantity of cases, but a particular pragmatic rule, called the sincerity condition (sometimes this principle is called Moore's principle). "The sincerity condition of asserting that p is that the speaker believes that p, that is, an assertion is sincere only if the speaker believes what he or she asserts. Accordingly, a sincere assertion that p counts as an expression of a belief with the same content, more precisely, it entails that the speaker holds the belief that p is the case." (see SCHMIDT 2014, 49). What is more, this principle is the constitutive rule for asserting

[5] In other words, the absurdity of this sentence is not derived from its internal contradiction what is expressed by the sentence, may possibly be true; the absurdity lies not in what the subject states, but in the conjunction of what the subject says with him or her saying what he or she says. Cf. WILLIAMS 1979, 141–2.

[6] The absurdity of Moore's sentence can be demonstrated in the following manner:

(0) $As(p) \rightarrow p \wedge B_{impl}(I, p)$ (Moore's principle)

(1) $As(p \wedge \neg B(I, p))$ (Moore's sentence)

by recognizing them in conjunction (0) as well as (1) we have:

(2) $As(p) \wedge \neg B(I, p) \wedge B_{impl}(I, p) \wedge B_{impl}(I, \neg B(I, p))$

see SCHMIDT 2014, 46; in the formulas presented by Schmidt only the "$\vdash$ sign was substituted (generally it is used in the meaning "It is the thesis that") by the "As" symbol as well as small corrections have been made.

and it is "grounded in there being an internal relation between each category of speech acts and the corresponding kinds of mental states. This analysis enables Searle to generalize Moore's Paradox for the other categories of speech acts." (cf. SCHMIDT 2014, 50). For Searle states "wherever there is a psychological state specified in the sincerity condition, the performance of the act counts as an expression of that psychological state. This law holds whether the act is sincere or insincere, that is whether the speaker actually has the specified psychological state or not. Thus to assert, affirm, state (that *p*) counts as an expression of belief (that *p*). To request, ask, order, entreat, enjoin, pray, or command (that A be done) counts as an expression of a wish or desire (that *A* be done). To promise, vow, threaten or pledge (that *A*) counts as an expression of intention (to do *A*). To thank, welcome or congratulate counts as an expression of gratitude pleasure (at H's arrival), or pleasure (at H's good fortune)" (see SEARLE 2011, 65). In summary, one can say that in both explanations of the reasons for the paradox it is assumed that language is the mean for reproducing representational contents; "I believe that *p*" and "I don't believe that *p*" relate to the adequate facts referring to the subject's mental state. The subject does not even have to utter Moore's sentences, but it is enough that he or she is convinced about it (it seems to be, in accordance with the condition of sincerity, implied by the act of ascertainment).

Two versions of Moore's paradox exist. The first one, discussed above, is called the 'omissive form,' because the subject that states it does not recognize a particular truth:

(O) "*p* and it is not the case *A* believes that *p*," (e.g. "I went to the pictures last Tuesday but I don't believe that I did") (cf. MOORE 1952, 542–3).

The second one, called the "commissive form," because when it states that *p*, the subject commits him or herself to error in its beliefs; the commissive form is formulated as follows:

(C) "*p* and *A* believes that it is not the case that *p*," (e.g. "I believe that he has gone out but he has not").[7]

In other words the omissive form is formulated as follows:

(O) $p \wedge \neg Bp$

a commissive form —

[7] *Russell's theory of descriptions*, w: SCHILPP 1944, 175-6; cf. also WILLIAMS 2013, 1117–38.

(C) $p \wedge B\neg p$.

These sentences do not express the same proposition, nor do they "commit the same absurdity in uttering both"; what is more, they are not connected to each other by relation of logical consequence (i.e. neither is the first a logical consequence of the other, nor does it go the other way). Williams emphasizes the difference in the 'source of absurdity' of each of these statements. "Normally, it is absurd for A to assert (O) because what is conjointly expressed and asserted, i.e. a belief that p and a lack of belief that p, is logically impossible. The absurdity in (C) is of a different kind. For normally, it is absurd for A to assert (C), not because what is conjointly expressed and asserted, i.e. a belief that p and a belief that it is not the case that p, is logically impossible, but because it is inconsistent (WILLIAMS 1979, 142).

In the initial phase of the considerations on the paradox its reason was presumed to be located (as indicated above) in the pragmatic context (utterances and acts of speech). Nevertheless, as Sorensen indicated, the paradox is present deeper than at the level of speech, namely at the level of thought; "if I silently believe in (O) or (C) then I seem no less absurd, yet what I believe might be true." (WILLIAMS 2013, 1119). Then it was assumed that the explanation of the source of the paradox should be sought at the level of beliefs; in the light of such an explanation one can attempt to demonstrate its absurdity at the level of asserting (*the priority of belief thesis*). In the analyses of the paradox with the use of nonclassical logics (and formal theories of belief change) presented below, it seems that this thesis is commonly recognized.

2 MOORE'S PARADOX
ACCORDING TO LOGIC OF KNOWLEDGE AND BELIEF

J. Hintikka was perhaps the first to analyze the problem in the language of epistemic logic (HINTIKKA 1962, 64–71). As Hintikka indicates, Moore's sentence which states that (O) 'p, but I don't believe that p' "strikes us as paradoxical." The paradoxicality of this sentence is connected, according to Hintikka, with its first-person character (however, as we shall see below, not only)[8]; if it were phrased: 'p, but he does not believe that p,' there would be

[8] The specifics of first-person utterances are broadly discussed in academic publications. It is

nothing out of the ordinary in that. In order to demonstrate its paradoxicality, it must be written down in the following way: 'I believe that the case is as follows: p but I don't believe that p,' that is:

(*) $B_a(p \wedge \neg B_a p)$.[9]

Hintikka demonstrates that in the belief system KD4 that he prefers[10] this sentence is impossible to defend. However, of course, to following sentence is defensible:

(**) $B_b(p \wedge \neg B_a p)$, as long as $a \neq b$.

Therefore the uttering subject cannot ascertain in relation to him or herself that he or she does not believe the sentence which is true, although there is nothing strange in the statement by such a subject that somebody also does not believe that this sentence is true.[11] Hintikka indicates the refutability of Moore's sentence with the aid of argumentation referring to his semantics of possible worlds, in which he assumes the existence of sets and

---

worth noticing, as linguists indicate, that two types of subjects can be discerned in epistemic sentences: the subject uttering an announcement with an epistemic verb and an epistemic subject which is the subject of an epistemic sentence containing an epistemic verb. Cf. DANIELEWICZOWA 2002.

[9] All epistemic first-person utterances have the character of utterances with the iteration of functors. And so: $B_a p$ states that a believes that p (e.g. 'Marek believes that p'), but only $B_a(B_a p)$ ('Marek believe that Marek believes that p,' where Marek is the name of the person speaking), may convey, as it seems, a first-person utterance "I believe that p"; interesting considerations on the topic of delving into iterations can be found in the article by R. Sorensen *Moore's problem with iterated belief* (= SORENSEN 2000). Sorensen notices that if we delve into the iteration (e.g. if we have a sequence of sentences: "The president is an adulterer," "I believe the president is an adulterer," "I believe that I believe the president is an adulterer," etc.) we receive ever weaker statements. This observation is rendered onto the considerations referring to Moore's problem. Using belief iterations brings out the asymmetry between the omissive and commisive version of the paradox. "The commissive version $p \wedge \neg B\neg p$ inflates into the usual unassertability and unbelievability of $p \wedge BBBBBBBBBB\neg p$. In contrast, the omissive version $p \wedge \neg Bp$ inflates into the unexpectedly plausible $p \wedge \neg BBBBBBBBBBp$. Many reasonable people believe that their higher-order beliefs peter out. Instead of always iterating without limit, some beliefs about beliefs and in less than ten iteration." (SORENSEN 2000, 29).

[10] System KD4 is the doxastic equivalent of the S4 system, i.e. normal modal logic consisting of the classical propositional calculus supplemented with the following axioms:

(K) $B(p \rightarrow q) \rightarrow (Bp \rightarrow Bq)$

(D) $Bp \rightarrow \neg B\neg p$

(4) $Bp \rightarrow BBp$ and Gödel's rule $\phi \vdash B\phi$.

[11] This example demonstrates the intensionality of doxastic contexts due to the subject that has beliefs; although (*) is a substitution of (**) which is defensible (but not *self-sustaining*), is an *indefensible* sentence *per se*.

model system as well as the accessibility relation governing these systems (Cf. HINTIKKA 1962, 69); let us assume here, in order not to introduce 'semantic machinery' a simpler, syntactic way of indicating the unacceptability of Moore's paradox (SORENSEN 2000). Namely, it is enough to accept the rule of distribution of the functor B over conjunction, rule of non-contradiction (BC) and respectively the rule of adding the B functor to the formula preceded by B (BI) for the paradox in its *omissive* version (these rules correspond to the particular semantic principles used by Hintikka in his argumentation against Moore's paradox) as well as the rule of omitting the B functor in the formula iterated by *commissive* version (which Hintikka did not analyze).

These are those principles according to Sorensen[12]:

$$\text{BKD: } \frac{B(p \wedge q)}{Bp \wedge Bq} \quad \text{BC: } \frac{Bp}{\neg B \neg p} \quad \text{BI: } \frac{Bp}{BBp} \quad \text{BE: } \frac{BBp}{Bp}$$

After such arrangements it is easy to indicate that Moore's paradox, on the grounds of adequate systems, leads to contradiction. The paradox in its *omissive* version becomes contradicted in the following way:

1.	$B(p \wedge \neg Bp)$	assumption;
2.	$Bp \wedge B\neg Bp$	BKD: 1
3.	Bp	OK:2
4.	BBp	BI: 3
5.	$B\neg Bp$	OK:2
6.	$\neg BBp$	BC: 5
7.	$BBp \wedge \neg BBp$	DK: 4, 6
8.	$\neg B(p \wedge \neg Bp)$	1, 7, *reductio ad absurdum*

The analogical proof of the *commissive version* presumes the use of rule BE (or axiom 4c).

It is worth drawing attention to the fact, as Hintikka assumes, that in order to show that Moore's sentence is impossible to defend, we do not have

[12] Instead of rules one can accept corresponding axioms, namely:
K. $B(p \wedge q) \rightarrow Bp \wedge Bq$
D. $Bp \rightarrow \neg B\neg p$
4. $Bp \rightarrow BBp$
4c. $BBp \rightarrow Bp$
5. $\neg Bp \rightarrow B\neg Bp$
5c. $B\neg Bp \rightarrow \neg Bp$.
The 4c version in the systems containing axiom T is simply the substitution of axiom T.

to refer to condition 4. i.e. we do not have to assume the transitivity of the alternativeness relation. It is enough to demonstrate that the sentence: (***) $(p \wedge \neg B_a p) \wedge B_a(p \wedge \neg B_a p)$ is impossible to defend and this, as such, seems to be trivial. In other words, if we paraphrase Moore's statement (***), the impossibility of defending it can be indicated even without showing the transitivity of the relation of alternativeness.[13]

Hintikka also constructs paradoxical Moore's sentences for the concept of knowledge:

1) "*p*, but I don't know that *p*" and for communicative contexts:
2) "*p*, but you don't know that *p*."

Hintikka's last considerations are interesting. Transmitting information to somebody, changes his or her knowledge and so it requires transgressing beyond Hintikka's basic assumption that we analyze somebody's knowledge (beliefs) in one state (Hintikka's logic is static — it defines the conditions of non-contradiction of the content of a given belief state). Hintikka states:

> The occasional oddity of (2) is not very difficult to account for. The first and foremost purpose of addressing a statement to somebody is to inform him of something. Addressing a statement to the person referred to by the announcement is in fact often called "letting *a* know something."Now uttering a sentence can only serve this purpose if it is possible for the person to whom it is addressed to know the truth of what he is being told. Thus if (2) is to serve the normal purpose of declarative second-person sentences, it must be possible for you to know what you are being told, that is, the sentence (3) "You know that the case is as follows: *p* but you do not know that *p*" must be defensible. But (53) is of the form (41) and therefore indefensible. In other words, (52) is an abnormal sentence in that it cannot ever be used to let the person to whom it is addressed know what this sentence expresses, for he cannot conceivably know it. For this reason, I submit, (52) is sometimes felt to be somewhat odd. (HINTIKKA 1962, 90).[14]

Moore's problem may be applied also to second-person utterances, but it surly does not apply to (as it was indicated at the beginning of our con-

---

[13] Nonetheless, there is a problem whether this formulation adequately reflects the content of Moore's sentence. It seems that it is stronger in (*); in (*) the truth value of *p* was only the content of belief, whereas in case of (***) the truth value of *p* is a given. From (***) immediately (*) is inferred, but not conversely; hence the argumentation for the indefensibility of (***) requires greater logical strength on the system's side.

[14] It is worth mentioning that there is an entire branch of dynamic logic dedicated to acquiring knowledge from announcements (logic of public announcements PAL) in which the basic assumption is that the announcement uttered by the broadcaster is true.

siderations) third-person ones. Hintikka explains that (as was indicated earlier) second-person utterances are not essentially different than first-person ones. The basis for the absurdity of Moore's second-person utterances is that the condition for communication is hearing the broadcast by the receiver and then the second-person message becomes (assuming the credibility of the informer) a first-person utterance. However, the third party (the epistemic subject) does not have to hear the message uttered and he or she does not have to know the truth value of p (cf. HINTIKKA 93–95). Of course in a first-person utterance the knowledge of the broadcaster is simultaneously the subject of the message and this is the source of the paradox. Therefore, as Hintikka expresses himself with Wittgenstein's words: 'my relation to my words is different from other people's relation to them'. In this sense "There is not a physical but a (so to speak) logical necessity of hearing what one is saying' if there were not, uttering (1) […] would not be any stranger than uttering 'p, but a does not know that p'. "p but a does not know that p." It is worth emphasizing Hintikka's opinion, who considers the logical solution to Moore's paradox to be more economical than other ones present elsewhere in academic literature, e.g. Moore's solution based on the fact that in the vast majority of cases we believe in what we say; uttering the second part (O) we fall into contradiction with what is implied by the utterance of the first part.

To conclude, it must be emphasized that the value of Hintikka's inquiries on the issue of Moore's paradox (but also in general in the entire logic of mental states) lies in that it shows how many epistemological issues can be resolved solely with the use of logical means. The elimination of Moore's problem is based exclusively on logical grounds, that is in principle — on consistency. One can abstract it for example from the thesis on the privileged access of the subject to his or her mental states.

Let us return once again to the issue of minimal conditions which a system of epistemic logic must fulfill in order to indicate the absurdity of Moore's statement. As we saw so far, the KD4 is sufficient to block the *omissive* version, and the KD4c — to block the *commissive* version of the paradox. Before we shall move on to the analysis of the systems that block the paradox, let us present the intuition behind the aforementioned axioms.

Axiom D called the principle of doxastic non-contradiction $\neg(Bp \wedge B\neg p)$ states that sentences Bp and $B\neg p$ are opposite sentences (they cannot simultaneously be true). Axiom 4. tends to be erroneously called 'positive introspection'; this name is elusive as this axiom states the positive omniscience of the subject with respect to his or her own beliefs and (as Hintikka indicated it in *Knowledge and Belief* it does not require the theorem of the privileged access to the content of one's consciousness.[15] In turn expression 4c. $(BBp \rightarrow Bp)$ states the infallibility of the subject with regards to its own beliefs (positive belief infallibility); 5. negative infallibility with respect to his or her own beliefs, a 5c — infallibility in relation to lack of beliefs (negative belief infallibility).[16]

In search of the weakest systems dismissing Moore's paradox, one can start from adding to the KD system formulas which simply are negations of the *commisive* (Ncm) and *omissive* (Nom) versions of Moore's sentence; then we have:

(Ncm) $Bp \rightarrow \neg BB\neg p$ (if the subject believes that p, then it is not so that he or she believes that he or she believes that not-p) and

(Nom) $Bp \rightarrow \neg B\neg Bp$ (if the subject believes that p, so it is not so that he or she believes that he or she does not believe that p). One thus receives the weakest augmentation of KD resistant to the paradox. Of course (Nom) is nothing else than 5c., i.e. infallibility within terms of the lack of belief (or else the infallibility in terms of recognizing one's lack of belief), however (Ncm), as Rieger notices, it was not granted a name for itself so far.[17] Rieger demonstrates, among others that:

a) K5c contains KDNom, i.e. blocking the omissive version blocks the commissive version of the paradox;

b) the opposite relation does not hold—one can avoid all of the commissive versions of the paradox and still be susceptible to the omissive version;

---

[15] The full names of the doxastic axioms were introduced by Witold Marciszewski. Cf. MARCISZEWSKI 1972.

[16] Brian F. Chellas calls the formula created by the conjunction of 4. and 4c. – 4! $(Bp \equiv BBp)$, and 5. and 5c. – 5! $(\neg Bp \equiv B\neg Bp)$; he claims that 4! is a thesis of every normal K5! logic as well as 4! and 5! are theses of every normal KD45 logic. Cf. CHELLAS 1980, 142. The analyses of the systems blocking Moore's paradox presented below are predominantly based on the work of Adam Rieger *Moore's paradox, introspection and doxastic logic* (= RIEGER 2015). We thank the anonymous reviewer for his comment stating that, as Johan van Benthem wrote, "Some weaken the logic in the argument still further. This is like turning down the volume on your radio so as not to hear the bad news. You will not hear much good news either" (VAN BENTHEM 2004, 95). The mere fact of choosing a weaker system is not enough to say that a paradox has been blocked." In Rieger's article we have indeed such a strategy. There are many strategies for changing a logical system when one faces some difficulties. Cf. HAACK 1978, 153–4.

[17] The following properties of the relation of doxastic accessibility correspond to formulas 4c, 5c (Nom) and Ncm:

4c.	$BBp \rightarrow Bp$	$\forall u \forall v(Ruv \rightarrow \exists w(Ruw \wedge Rvw))$	density
c.	$B\neg Bp \rightarrow \neg Bp$	$\forall u \exists v(Ruv \wedge \forall w(Rvw \rightarrow Ruw))$	
Ncm	$Bp \rightarrow \neg BB\neg p$	$\forall u \exists v(Ruv \wedge \exists w(Rvw \wedge Ruw))$	

c) K4! is strong enough in order to avoid both paradoxes; what is more: K4! may be weakened to KD4; K4c alone is only enough to eliminating the commissive version, but not the omissive version of paradox. All relations between systems are illustrated by the following diagram:

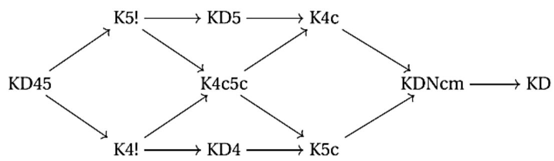

Image: *Relations between systems* (cf. RIEGER 2015, 221).

3. MOORE'S PARADOX IN THE AGM THEORY OF BELIEF CHANGE

Moore's paradox can also be reproduced in the AGM theory of belief change.[18] The minimal means which deter Moore's paradox were shown above. In this part we shall demonstrate that on the grounds of the AGM theory the attempt to express beliefs referring to one's own beliefs encounters difficulties that are (in principle) insurmountable.

The AGM belief change theory is probably the most known popular approach to belief changes.[19] It is assumed that the basic concept of the so-called belief state represented by a consistent set of sentences is closed under classic consequence. This state is in a sort of balance (*epistemic equilibrium*), precipitated by new information, represented by this sentence. The change of the epistemic state may rely on adding a sentence to it (expansion), removing a sentence — contraction and replacing a sentence by its negation — revision. These changes are described with the aid of the following set of axioms:

[18] I presented this issue in greater detail in article *Dlaczego logika zmian przekonaniowych „nie lubi" zdań introspekcyjnych* [Why the change belief logic "dislikes" introspective sentences] (= LECHNIAK 2010).

[19] For the most philosophical presentation of theory, see GÄRDENFORS 1988; cf. also an analysis of various aspects of AGM in LECHNIAK 2011.

AXIOMS FOR EXPANSION:

(K+1) K_ϕ^+ is set of beliefs	Closure
(K+2) $\phi \in K_\phi^+$	Success
(K+3) $K \subset K_\phi^+$	Inclusion
(K+4) If $\phi \in K$, then $K_\phi^+ = K$	Lack of change
(K+5) If $K \subset H$, then $K_\phi^+ \subset H_\phi^+$	Monotony

(K+6) For every set of beliefs and every sentence ϕ ,
K_ϕ^+ is the smallest set of beliefs, which fulfills conditions (K+1) – (K+5).

AXIOMS FOR CONTRACTION:

(K-1) For every sentence ϕ and any random set of beliefs K, K_ϕ^- is a set of beliefs.	Closure
(K-2) $K_\phi^- \subset K$	Inclusion
(K-3) If $\phi \notin K$, then $K_\phi^- = K$	Emptiness
(K-4) If it not so that $\vdash \phi$, then $\phi \notin K_\phi^-$	Success
(K-5) If $\phi \in K$, then $K \subset (K_\phi^-)_\phi^+$	Recovery
(K-6) If $\vdash (\phi \equiv \psi)$, then $K_\phi^- = K_\psi^-$	Extensionality
(K-7) $(K_\phi^- \cap K_\psi^-) \subset K_{(\phi\wedge\psi)}^-$	
(K-8) If $\phi \notin K_{(\phi\wedge\psi)}^-$, then $K_{(\phi\wedge\psi)}^- \subset K_\phi^-$	

AXIOMS FOR REVISION:

(K*1) For every sentence A and any given set of beliefs K, K_ϕ^* is set of beliefs.	Closure
(K*2) $\phi \in K_\phi^*$	Success
(K*3) $K_\phi^* \subset K_\phi^+$	Inclusion
(K*4) If $\neg\phi \notin K$, then $K_\phi^+ \subset K_\phi^*$	Saved without changes
(K*5) $K_\phi^* = K_\perp$ then and only then $\vdash \neg\phi$	Consistency
(K*6) If $\vdash (\phi \equiv \psi)$, then $K_\phi^* = K_\psi^*$	Extensionality
(K*7) $K_{(\phi\wedge\psi)}^* \subset (K_\phi^*)_\psi^+$	
(K*8) If $\neg\psi \notin K_\phi^*$, then $(K_\phi^*)_\psi^+ \subset K_{(\phi\wedge\psi)}^*$	

Believing that ϕ is represented by the proposition that $\phi \in K$; usually (without introspection) beliefs relate to the world. The main difficulty with introspective sentences in the AGM theory relies on the issue that the permission for sentences about the world in a set of beliefs to be accompanied by sentences about these sentences immediately leads to the generation of Moore's paradox. We are dealing with a situation where the cognizing subject utters a particular sentence simultaneously stating that it does not believe that things are in the way its utterance states. Since the subject utters such a sentence, he or she should believe that things are the way he or she states, i.e. $B_a(p \wedge \neg B_a p)$.

The reasoning leading to proving Moore's sentence in AGM is the following (cf. LINDSTRÖM & RABINOWICZ 1997, 138–139):

Let us assume that the subject does not believe sentence ϕ nor its negation $\neg\phi$ (he or she does not have an opinion on a given topic)

1.	$\phi \notin K \wedge \neg\phi \notin K$	assumption
2.	$\neg B\phi \in K$	assumption (I accurately believe that I do not believe ϕ)
3.	$\neg\phi \notin K \rightarrow K^*_\phi = K^+_\phi$	(K*3), (K*4)
4.	$K^*_\phi = K^+_\phi$	1, 3
5.	$K \subset K^+_\phi$	(K+3)
6.	$\neg B\phi \in K^+_\phi$	5, 2
7.	$\neg B\phi \in K^*_\phi$	4, 6
8.	$\phi \in K^*_\phi$	(K*2)
9.	$(\neg B\phi \wedge \phi) \in K^*_\phi$	7, 8

Assuming the theorem about positive omniscience in terms of introspection $B\phi \rightarrow BB\phi$[20] we get the contradiction of expression $(B\phi \wedge \neg B\phi) \in K^*_\phi$ with (K*5).

We can see that in order to prove Moore's sentence we do not have to deal with revision, but this sentence is a result of using (based on the commensuration thesis)[21] of a seemingly unproblematic operation of expansion

[20] It is practically always assumed in case of beliefs that at least the KD4 system and most often an even stronger system, i.e. KD45.

[21] Id est, the ascertainment that every belief revision is reducible to the compilation of contraction and expansion which is stated by what is called Levi's definition:

(Def. *) $K^*_A = (K^-_{\neg A})^+_A$.

(which is a simple logical extension of the input set of beliefs $K^+_\phi = Cn(K \cup \{\phi\})$. The reasoning for expansion is the following:

1.	$\phi \notin K$	assumption
2.	$\neg B\phi \in K$	assumption (I rightfully believe that I do not believe ϕ)
3.	$K \subset K^+_\phi$	(K+3)
4.	$\neg B\phi \subset K^+_\phi$	2, 3
5.	$\phi \in K^+_\phi$	(K+2)
6.	$(\phi \wedge \neg B\phi) \in K^+_\phi$	4, 5

This last fact is worth emphasizing. For in AGM, Moore's paradox is not connected with revision but with adding to the body of knowledge, because we assume that the input contains $\neg B\phi$ and does not contain ϕ new information. The negative introspective sentence, therefore, 'resides' in the set of beliefs *K*, and therefore also in the K^+_ϕ set (and every subsequent belief state). One must also add then Moore's conjunction $\phi \wedge \neg B\phi$ to set K^+_ϕ. Therefore, the reason for the difficulty is the lack of a 'mechanism' for removing the negative introspective sentence as a result of adding the subject of this introspective negative proposition to the set of beliefs, i.e. sentence ϕ.[22]

The first attempt by Lindström and Rabinowicz to remove this difficulty was limiting the range of influence of the axiom (K*4)[23] exclusively to Boolean sentences (i.e. sentence about the external world, without the B functor). They indicated the need for weakening the axiom to the form of the 'weak preservation' axiom:

If $\neg\phi \notin K$ and ψ is a Boolean formula and $\psi \in K$, then $\psi \in K^*_\phi$.[24]

[22] "When the introspective agent learns more about the world (and himself) then the reality that he holds beliefs about undergoes change. But then his introspective (higher-order) beliefs have to be adjusted accordingly." (LINDSTRÖM & RABINOWICZ 1999b).

[23] It seems that axiom (K*4) which states together with (K*3) that expansion is a special type of revision, similarly to the axiom of recovery for contraction the neuralgic point of revision axioms.

[24] The reasoning can be reconstructed in the following way: $\neg\phi \notin K \to \forall\psi(\psi \in K^+_\phi \to \to \psi \in K^*_\phi)$ (K*4). But $\forall\psi(\psi \in K \to \psi \in K^+_\phi)$ (K+3), thus we receive $\neg\phi \notin K \to \to (\psi \in K \to \psi \in K^*_\phi)$ and only this formula is weakened here. Cf. LINDSTRÖM, RABINOWICZ 1997, 138–139.

This solution, as one can see, relies on prohibiting the introduction of introspective sentences to AGM formulas, i.e. it can seem to be a typical *ad hoc* solution for difficulties through blocking particular properties of the subject (the set of beliefs) which lead to these difficulties.[25] Another attempt to remove the difficulty with Moore's sentence is changing the definition of expansion and revision, heading, therefore, towards the limitation of the 'cautious' character of the last operation. According to Levi's definition revision is deconstructible to a sequence of contractions and expansions, i.e.

---

[25] In the system of belief change one can 'prove' several paradoxes analogous to Moore's. All of them have a similar structure, i.e. they are based on the assumption that the negative introspective sentence belongs to the set of beliefs or, as in the case of Fuhrman's paradox, that it may be added onto it. Therefore we have a so-called (*strong paradox*) formed in the following way: $\neg\phi \notin K \wedge \neg B\phi \in K \rightarrow K^*_\phi = K_\perp$. In the argumentation leading up to this paradox the axiom of preservation is assumed as well as the principle of positive introspective omniscience:

1.	$\neg\phi \notin K$	assumption
2.	$\neg B\phi \in K$	assumption
3.	$\neg\phi \notin K \rightarrow (\psi \in K \rightarrow \psi \in K^*_\phi)$	(K'*4)
4.	$\neg B\phi \in K^*_\phi$	1, 2, 3
5.	$\phi \in K \rightarrow B\phi \in K$	Positive introspection
6.	$\phi \in K^*_\phi$	Succes
7.	$B\phi \in K^*_\phi$	5, 6
8.	$B\phi \wedge \neg B\phi \in K^*_\phi$	4,7
9.	$K^*_\phi = K_\perp$	8

If we, in turn, interpret the deontic AGM theory in the spirit of the expressivist concept of norms and this was, indeed, one of the sources of the AGM theory, we shall receive the deontic analogate of Moore's paradox; the argumentation is analogous to the one presented above: If we replace B by $\neg E \neg$ (epistemic possibility), and we use the functor of permission (P) instead of E and we adopt the square of oppositions for deontic concepts, we can obtain a counterpart of Moore's problem for permission:

1.	$A \notin K$	hyp
2.	$P\neg A \in K$	hyp (strong permission)
3.	$K \subset K^+_A$	(K+3)
4.	$P\neg A \subset K^+_A$	2, 3
5.	$A \in K^+_A$	(K+2)
6.	$(A \wedge P\neg A) \in K^+_A$	4, 5

$K^*_\phi \equiv_{df} (K^-_{\neg\phi})^+_\phi$. When $\neg\phi \notin K$, contraction on account of $\neg\phi$ is empty and revision is reducible to expansion. And this situation, indeed, takes place in the case of paradoxes. Revision fulfilling the preservation axiom is 'too cautious' because expansion is cumulative. Because of this the substitution of 'regular' expansion with *cautious expansion* is proposed; before we expand the set of beliefs with ϕ, we should make sure that we abandoned the conviction that we do not believe ϕ; cautious expansion is not entirely cumulative. Hence the definition:

$$K^\oplus_\phi =_{df} (K^-_{\neg B\phi})^+_\phi$$

The cautious expansion of set K on account of ϕ ($K^\oplus_\phi$) relies on that we initially *contract* this set due to $\neg B\phi$ (i.e. we remove the introspective sentence) and only later do we *expand* K on account of ϕ. This way, in accordance with Levi's definition, we attain a new definition of 'cautious revision' (cf. LINDSTRÖM & RABINOWICZ 1997, 147 f.):

$$K^\otimes_\phi =_{df} (K^-_{\neg\phi})^\oplus_\phi$$

However, the definition of cautious revision does not impact the solution for the problem of introspective sentences, because its 'core' is the property of cumulative expansion. At this moment I would like to dedicate several remarks to this last operation.

It is necessary to underline the fact that cautious expansion is not monotonic. This is caused by the contraction which constitutes it (which, similarly to revision, is also nonmonotonic). The nonmonotonicity of cautious expansion can be demonstrate in the following way:

Let $\neg B\phi, \neg\psi \in K$ as well as $\neg B\phi, (\psi \to B\phi) \in H$. Therefore also $\neg\psi \in H$. Now after a cautious expansion on account of ϕ both $\neg\psi$ as well as $\phi \in K^\oplus_\phi$, however, $(\psi \to B\phi), \phi \in H^\oplus_\phi$. Thus there is a belief ($\neg\psi$) which belongs to $K^\oplus_\phi$, but does not belong to $H^\oplus_\phi$ and therefore it is not so that $K^\oplus_\phi \subset H^\oplus_\phi$.

Nonmonotonicity is flawed in relation to the expansion operation; it cannot be treated as a usual set theory extension of the set of beliefs. Yet, this introduction of a new concept of cautious expansion is also an *ad hoc* solution. At each attempt of adding information, the state of beliefs should be searched on account of introspective sentences and cleared of such sentences. This of course could generate difficulties connected with the non-monotonicity of cautious expansion. Although this does not refer only to negative introspective sentences, but also to positive ones.[26]

It seems that the problem lies deeper. An introspective sentence is occasional by nature. This means that it informs about the *current* state of beliefs (cf. JUDYCKI 2002).[27] If the fact that introspective sentences refer to the current state does not find its representation in the language of the theory, then getting Moore's sentences will always be possible. The rational introspective cognizing subject *notes* in his or her consciousness the consequence of his or her introspective data with the aid of such phrases as: 'I believed in an earlier state (in moment t_1) that p,' 'I did not know earlier (though now I know) that p,' etc. Therefore, we can have in a given belief state (in moment t_2 such that $t_2 > t_1$) *collision free* sentences $B(t_1)p$ and $B(t_2)\neg p$; we simply distinguish the moments, to which our introspective knowledge relates to. However, the AGM theory compares only two neighboring belief states without the linguistic means at their disposal, in order to express the moments to which the introspective sentences are referring (*one–shot theory*); it refers to change of the belief state in isolation from other changes (it does not permit the iteration of belief changes). At the same time the *world*, which the beliefs refer to, is changed in such a manner that the introspective sentence referring the earlier belief state are falsified in the subsequent state.[28] It is worth recalling the traditional distinction: *reflexio in actu exercito* (*conscientia concomitans*) and *reflexio in actu signato*. It seems that no subject is able to utter in good faith Moore's sentence based on *reflexio in actu exercito* (if it has no disturbances

---

[26] Let both $\phi \in K$, as well as $B\phi \in K$. Then $\neg\phi \in K^*_{\neg\phi}$ and $B\phi \in K^*_{\neg\phi}$. Therefore also $(\neg\phi \wedge B\phi) \in K^*_{\neg\phi}$; we get Moore's sentence again.

[27] Every functor of iteration B (K) on the logic's systemic level is an expression of introspection, because the subject of beliefs (as long as both B are the expression of the same subject) are the subject's own beliefs (Smullyan drew attention to the fact that an analogous case occurs with Gödelian sentences, which are auto-reflexive, when he applied the perspective of doxastic logic to Gödel's problems). In turn the theses where one can find formulas with a singular functor B together with iterated B ones express the properties of 'logical introspection' (e.g. infallibility in terms of self-diagnosis, self-diagnostic-omniscience, etc.). To what extent beliefs are understood explicitly and to what extent 'implicitly' is another issue.

[28] As was pointed out by Katsuno and Mendelzon, followed by Friedman and Halpern: 'the AGM's postulates assume that the world is static; to capture this, we assume that the environment state does not change over time.' AGM describes the change of beliefs relating to the unchanging world. Cf. FRIEDMAN & HALPERN 1999.

of consciousness — simply co-consciousness, apart from exceptional cases, is accompanied by a stream of experiences); otherwise it is in *reflexio in actu signato*, but in this reflection the introspective sentence relates to that, which the subject has experienced, one of the subject's prior experiences, as it requires the intentional targeting of the act of consciousness in that case (if the subsequent belief states were to be symbolized as $K_1, \ldots, K_k$) the introspective sentence belongs to K_{k+1}, and the sentence which is its subject belongs to K_k, but then Moore's paradox may not occur.

The demonstrated difficulties with Moore's sentence are partially resolved by a more complicated formal construct proposed by Lindström and Rabinowicz (Cf. LINDSTRÖM & RABINOWICZ 1999a, 353–85; 1999b). The purpose is to block the possibility of the occurrence of Moore's sentences by way of adequately modifying semantics for the revision operation. In regular, non-introspective contexts expansion and contraction operations decrease the number of possible worlds which are compatible with the subject's beliefs in a given doxastic state (expansion) and increase the number of these worlds (contraction). Introspective sentences complicate this image.

The solution proposed by Lindström and Rabinowicz is, indeed, going in that direction and it is headed to separating these two elements of the beliefs which the introspective subject has.

> When the introspective subject acquires a new piece of information, his or her belief state undergoes a change. In this new state, his or her beliefs refer to both the old belief state and the new one. In general, it is necessary to differentiate between the state in which beliefs are evaluated (*point of evaluation*) and the state in which particular things are recognized (the state which the beliefs refer to, *point of reference*). (Cf. LINDSTRÖM & RABINOWICZ 1999b, 18–21).

Hence the basic assumption of semantics is the differentiation of two components in every state x: the 'worldly' component $w(x)$ and the doxastic component $d(x)$; therefore $x = (w(x), d(x))$. The component of the doxastic states are determined by two relations of accessibility between states:

a) The relation of accessibility which represents the subject's beliefs: $b(x, y, z)$ if and only if z is compatible with what the subject believes at point x about y.
b) The relation of accessibility connected with the revision operation: $R(*\alpha)(x, y, z)$ if and only if z is a state that the subject may reach from state x as a result of a revision (conducted) on account of sentence α of the belief state relating to y.

Additionally, since revision is a purely doxastic action, it does not influence the states of the external world, that is why the transition from state x to state z does not have an impact on the worldly component, i.e. If $R(*\alpha)(x,y,z)$, then $w(x)=w(z)$.

After introducing the differentiation between the evaluation point and the reference point the description of the truth value becomes altered; instead of speaking about the truth value at point x, the concept of the truth value at a given point in reference to another point is introduced — the same sentence can be true in relation to one point, but not be true in relation to another one (formula α is true in point x (evaluation point) because of point y (point of reference): $x,y\vDash\alpha$). For every α and every $y\in U$, $||\,\alpha\,||_y=\{x\in U : x,y\vDash\alpha\}$ is a proposition expressed by α on account of reference point y. The proposition of $P\subset U$ is 'worldly,' when it is closed under 'worldly equivalence,' i.e. always when $x\in P$ as well as $w(x)=w(y)$, then $y\in P$.

In two-dimensional semantics, apart from regular conditions for classic functors, we have two conditions for modality:

a) $x,y\vDash B(\alpha)$ if and only if for every $z,$ such that $b(x,y,z)$, $z,y\vDash\alpha$.

Sentence $B(\alpha)$ is true at point x with respect to y, when for every z such that z is doxastically possible in reference to everything that the subject recognizes at point x about y, sentence α is true at point z on account of y.

b) $x,y\vDash[*\alpha]\beta$[29] if and only if for every z such that $R(*\alpha)(x,y,z), z,y\vDash\beta$.

Sentence $[*\alpha]\beta$ is true in point x because of point y, when for every z accessible from point x as a result of revision caused by α of the state of beliefs referring to point y, β is true in z because of y.

Subsequently a new functor † is introduced, thanks to which one can identify the current evaluation point with the point of reference:

$x,y\vDash\dagger\alpha$ if and only if $x,x\vDash\alpha$.

The procedure allows to differentiate two types of beliefs:

— later belief about the output state (i.e. the state that occurs before the doxastic operation); condition: $x,x\vDash[*\alpha]B\beta$ refers to this situation, as well as

[29] Expression $[*\alpha]\beta$ of dynamic doxastic logic is read: "After revision, on account of sentence α, the state described by β" occurs; whereas phrase $[*\alpha]B\beta$ should be read: "After revision, on account of sentence α, the fact that the subject recognizes sentence β" occurs, which in the language of AGM relates to phrase $\beta\in K*\alpha$.

— later belief about the later state (when the subject is already after the doxastic action) — $x, x \vDash [{*}\alpha] \dagger B\beta$.

Formula α is true at point x if and only if, when $x, x \vDash \alpha$. In other words, α is true at point x when proposition $\| \alpha \|_x$ expressed by α on account of x is true solely at point x. The formula is generally valid in model **M** $M \vDash \alpha$, when α is true at point x, and α is generally valid in a strong sense if and only if for every pair of points $, x, y >$ in model **M**, $x, y \vDash \alpha$.

Formula α is *regular*, when its logical value does not rely on a reference point, i.e.

For every x, y, z: $x, y \vDash \alpha$ if and only if $x, z \vDash \alpha$.

It is worth pointing to the fact that the arrangements above allow Lindström and Rabinowicz to demonstrate that the formula which corresponds to Moore's sentence (in the language of AGM):

$$(\neg\alpha \notin K \wedge \neg B\alpha \in K) \rightarrow (\alpha \wedge \neg B\alpha) \in K * \alpha\,)$$

(1) $\neg B\neg\alpha \wedge B\neg B\alpha \rightarrow [{*}\alpha](B\alpha \wedge B\neg B\alpha)$

is not paradoxical anymore, although in a strong sense it is generally valid. However, the following formula is not generally valid

(2) $\neg B\neg\alpha \wedge B\neg B\alpha \rightarrow [{*}\alpha]\dagger(B\alpha \wedge B\neg B\alpha)$.

Expression (1), therefore, refers to that, what the subject after acquiring information α would assume about the state before revision (the point of reference is different than the point at which we conduct the evaluation), while (2) refers to that, what the subject shall know about the state received after the revision (reference and evaluation points are equated).[30]

4. FINAL REMARKS

As we stated above the aim of logical analysis of Moore's paradox is different than the aim of epistemologist's analysis. Logical analysis is focused on finding minimal systems in which the paradox is stopped while philosophers seek what specific features of knowledge (belief) that allow the paradox.

Now, let us try to collect and compare the abovementioned attempts of resolving or removing the paradox.

[30] However, Lindström and Rabinowicz indicate that the new construct is also not devoid of difficulties. Admittedly, it allows to determine the subject's *later* beliefs about its *earlier* state, but it does not allow to determine the subject's *later* beliefs about his or her *earlier* state.

1) Moore's paradox (both in its *omissive* and *commisive* versions), despite broad epistemological investigations, is relatively easy to block in the case of so-called static epistemic (doxastic) logic; the weakest systems are K4!, K5c and KDnom. The cost of this solution is high, because we lose some of the laws which describe rational beliefs.

2) The situation gets worse with belief change theories, in case of which Moore's paradox as well as the very existence of introspective sentences causes practically insurmountable difficulties. Various attempts of securing the system do not guarantee satisfactory solutions. The following approaches differ in their advantages and shortcomings:

a) limiting the range of influence of the axiom (K*4) exclusively to Boolean sentences: The paradox is removed by blocking particular properties of the set of beliefs which lead to these difficulties; however, we want to have introspective beliefs;
b) redefinition of the operations of contraction and expansion: The paradox is blocked, but the new operation — cautious expansion — is non-monotonic.

3) The situation, however, is not hopeless when it comes to the possibility of constructing a theory of belief change with introspective sentences not containing the paradox; a specific rescue measure can be found in these logics with the use of which one can express both change of beliefs and introspective sentences. As for the paradox itself: securing the belief system from the possibility of the paradox occurring requires discerning the set of beliefs from the set of sentences about beliefs at a semantic level. It is possible in the case of dynamic doxastic logics with their modal language in which one can distinguish two kinds of operators: an operator of belief and operators of change.

REFERENCES

BALDWIN, Thomas (ed.). 1993. *G.E. Moore: Selected writings*. New York, London: Routledge.

CHELLAS, Brian F. 1980. *Modal logic*. Cambridge: Cambridge University Press.

FRIEDMAN, Nir, and Joseph Y. HALPERN. 1999. "Belief revision: a critique". *Journal of Logic, Language and Information* Vol. 8, 401–20.

DANIELEWICZOWA, Magdalena. 2002. *Wiedza i niewiedza. Studium polskich czasowników epistemicznych*. Warszawa: Uniwersytet Warszawski.

GÄRDENFORS, Peter. 1988. *Knowledge in flux. Modelling the dynamics of epistemic states*. Cambridge, MA: A Bradford Book, The MIT Press.

GREEN, Mitchell S., & John N. WILLIAMS. 2007. *Moore's Paradox: New essays on belief, rationality, and the first person*. New York: Oxford University Press.

HAACK, Susan. 1978. *Philosophy of logics*, Cambridge: Cambridge University Press.

HINTIKKA, Jaakko. 1962. *Knowledge and belief*. Ithaca: Cornell University Press.

JUDYCKI, Stanisław. 2002. "Introspekcja jako problem filozoficzny." *Roczniki Filozoficzne* vol. 50, issue 1, 263–301.

LECHNIAK, Marek. 2010. "Dlaczego logika zmian przekonaniowych „nie lubi" zdań introspekcyjnych." In Wojciech SUCHOŃ, Irena TRZCIENIECKA-SCHNEIDER & Dominik KOWALSKI (eds.). *Argumentacja i racjonalna zmiana przekonań*. (Dialogikon, t. XV), 141–58. Kraków: Wydawnictwo Uniwersytetu Jagiellońskiego.

LECHNIAK, Marek. 2011. *Przekonanie i zmiana przekonań*: *Analiza logiczna i Filozoficzna*. Lublin: Wydawnictwo KUL.

LINDSTRÖM, Sten, & Wlodek RABINOWICZ. 1997. "Extending dynamic doxastic logic: Accommodating iterated beliefs and Ramsey conditionals within DDL." In Lars LINDAHL, Paul NEEDHAM, & Rysiek ŚLIWIŃSKI (eds.). *For good measure: Philosophical essays dedicated to Jan Odelstad on the occasion of his fiftieth birthday*. (Uppsala Philosophical Studies, 46), 126–53, 138–39. Uppsala: Department of Philosophy, University of Uppsala.

LINDSTRÖM, Sten, & Wlodek RABINOWICZ. 1999a. "DDL unlimited: dynamic doxastic logic for introspective agents." *Erkenntnis* vol. 50: 353–385.

LINDSTRÖM, Sten, & Wlodek RABINOWICZ. 1999b. "Belief change for introspective agents." In Sören HALLDÉN, Bengt HANSSON, Wlodek RABINOWICZ, & Nils-Eric SAHLIN (eds.) *Spinning ideas: Electronic essays dedicated to Peter Gärdenfors on his fiftieth birthday*. Lund: Department of Philosophy, Lund University. https://www.lucs.lu.se/spinning/categories/dynamics/Lindstrom_Rabinowicz/index.html

MARCISZEWSKI, Witold. 1972. *Podstawy logicznej teorii przekonań*. Warszawa: PWN.

RIEGER, Adam. 2015. "Moore's paradox, introspection and doxastic logic." *Thought* 4: 215–27.

SCHILPP, Paul Arthur (ed.). 1944. *The philosophy of Bertrand Russell*, Evanston (IL): Northwestern.

SCHILPP, Paul Arthur (ed.). 1952. *The philosophy of G.E.Moore*. New York: Tudor Publishing Company.

SCHMIDT, Ulla. 2014. *Moore's paradox. A critique of representationism*. Berlin: De Gruyter.

SEARLE, John. 2011. *Speech acts: An essay in the philosophy of language*. Cambridge: Cambridge University Press

SORENSEN, Roy. 2000. "Moore's problem with iterated belief." *The Philosophical Quarterly* vol. 50, no. 198: 28–43.

VAN BENTHEM, Johan. 2004. "What One May Come to Know." *Analysis* vol. 64, no. 2: 95–105.

WILLIAMS, John N. 1979. "Moore's paradox: one or two?"*Analysis* vol. 39, no. 3: 141–42.

WILLIAMS, John N. 2013. "Moore's paradox and the priority of belief thesis." *Philosophical Studies* 165 (3): 1117–38.

ONCE MORE ABOUT MOORE'S PARADOX IN EPISTEMIC LOGIC AND BELIEF CHANGE THEORY

Summary

In this article, it was first presented Moore's paradox *per se* and after the author focused on the logical perspective — at first he analyzed these considerations in the field of so-called standard epistemic logic and after on the formal theory of belief change.

JESZCZE RAZ O PARADOKSIE MOORE'A W LOGICE EPISTEMICZNEJ I TEORII ZMIAN PRZEKONANIOWYCH

S t r e s z c z e n i e

W niniejszym artykule najpierw został zaprezentowany paradoks Moore'a *per se*, a następnie autor skupił się na perspektywie logicznej, analizując wpierw problem w zakresie tak zwanej standardowej logiki epistemicznej, a potem formalnej teorii zmian przekonaniowych.

Key words: Moore's paradox; epistemic logic; belief change theory.
Słowa kluczowe: paradoks Moore'a; logika epistemiczna; teoria zmian przekonaniowych.

Information about Author: Dr hab. MAREK LECHNIAK, Prof. at KUL — Department of Logic, Faculty of Philosophy, John Paul II Catholic University of Lublin; address for correspondence: Al. Racławickie 14, 20-950 Lublin; e-mail: lechmar@kul.pl