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LANGUAGE AS A NECESSARY CONDITION FOR COMPLEX MENTAL CONTENT: A REVIEW OF THE DISCUSSION ON SPATIAL AND MATHEMATICAL THINKING

1. INTRODUCTION

In this text, we critically discuss the thesis that natural language is a unique cognitive tool which integrates mental representations coming from other cognitive subsystems. The view holds that mastering natural language plays the central role in the uniquely human ability of flexible conceptual manipulation, e.g. moving concepts from one domain to another or integrating many concepts into one. The position is located within the version of the modularist framework which assumes a moderate view on the extent of modularity, somewhere in between the peripheral modularist view and global modularism (CARRUTHERS 2002, 2012; HERMER-VAZQUEZ, SPELKE, & KATSNELSON 1999; HESPOS & SPELKE 2004; IZARD, SANN, SPELKE, & STRERI 2009; LAURENCE & MARGOLIS 2008; LI & GLEITMAN 2002; SPELKE 2003).

On this view, natural language's role in the cognitive system is to provide a medium for the kind of thinking that integrates data coming from domain-specific conceptual modules (BERMÚDEZ 2003; CARRUTHERS 2012; GUMPERZ & LEVINSON 1996, 145–176). Language faculty is thus a system of inter-modular communication, integrating contents distributed across different mo-

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dules. Moreover, not only does it make it possible to express information from those modules, but it is also capable of significantly changing and augmenting that information (CARRUTHERS 2003a; SPELKE 2003).

The integrative function of language sketched above is nothing else but a specific statement of the general view that language helps form specifically human kinds of thought. Proponents of the view usually hold that what is specific to human mind is the *ability to combine* mental representations, rather than some uniquely human cognitive module (CARRUTHERS 2002; HERMER & SPELKE 1996; cf. LAURENCE & MARGOLIS 2008; SPELKE 2003). The natural flexibility and creativity of human thought is held up as the thing that distinguishes us from other animals. This flexibility or creativity consist in, for example, the fact that we can conceptually combine the radiance of the sun with the radiance of a woman's beauty, or our fate with the fate of a reed bending in the wind. It also shows in the fact that we deploy superordinate concepts that can subsume other, separate concepts, and that we often ponder over the criteria of application of the given concept to a particular instance. Arguably, this is made possible by our ability to integrate contents from various cognitive modules. Spelke discusses the role of language in that integrative ability:

> Natural languages provide human with a unique system for combining flexibly the representation they share with animals. The resulting combinations are unique to humans and account for unique aspects of human intelligence. (SPELKE 2003, 291)

Language, on that view, is given a special significance, as it is constitutive of the uniquely human kinds of thinking. The human, apart from domain-specific cognitive modules, is said to possess the unique combinatorial system of language. Contents coming from various sources are transduced into the linguistic format, which effectively means that a considerable part of human thought happens in language, in sequences of sentences of natural language represented in the mind (CARRUTHERS 1996, 225–276).

2. THEORETICAL ASSUMPTIONS AND THE CONTEXT OF THE PROPOSAL

The proponents of the language-as-an-integrator hypothesis (henceforth LIH) stress that language is not merely a means for expression of information from different modules. Although it is one of its function, the claim is much stronger — information from the modules can be transduced into a common format thanks to language, which results in the kind of thought which would not exist without language. In other words, language is a necessary condition for certain kinds of thought to exist. Thus, language is not a mere tool for making thoughts public, but a constituent of certain thoughts, which cannot be expressed by other formats. One cannot, on this view, push the language into the background, as its role is central to the human cognitive system. Neither can the function of language be narrowed down to abilities explicitly dependent on linguistic skills, nor ascribed only to the workings of long-term memory and planning. External and internal verbalization cannot be marginalized.

The essence of LIH is such that language (especially its formal aspects) is an integrator of contents from domain-specific modules and creator of amplified information that goes beyond their limitations. The main research methodology by the proponents of the view is to find content that comes from modules that are independent from language and common to a wide number of other species. Then, researchers go on to show how acquiring language makes possible to combine contents from these modules and "generate" kinds of thoughts that are specific to humans and which depend on language. The method is nicely illustrated by two examples: spatial thinking and mathematical thinking. Spatial thinking is said to depend on combining information about the geometrical structure of the environment and information about the properties of particular elements in the environment. Mathematical thinking is similarly claimed to rely on the integrative function of language that enables the acquisition of the concept of discrete numerosity.

In the current text, we present the two research areas mentioned above. However, our main focus will fall on mathematical thinking and this will also be the point of our critique. We will illustrate how linguistic competence leads to transcending the representational limitations of the modular capacities for small number representation and approximate numerosity representation. It becomes possible to represent higher numbers discretely because language makes it possible to represent that adding one to any number gives the next higher number. The claim is that going beyond the ability to represent numerosities only approximately is connected with language acquisition. It has been demonstrated that we need digits represented linguistically to think about large numbers. During arithmetic calculations, information that is kept in short term memory has phonetic format (Baddeley 1986; Kurcz 1976; Nęcka, Orzechowski, & Szymura 2006; Smith 1999). Further mathematical skills—such as adding, comparing numbers etc. —

will effectively depend on language too, as the basic understanding of the number on which they rely is allowed by language in the first place. The mathematical mind of the human undoubtedly surpasses those of other animals. The view locates itself somewhere in between the empiricism that identifies every thought with language and the view that claims the reverse, that language acquisition hinges on our preexisting mathematical capacities (LEIBNIZ 1976).

The moderate modularity espoused by the proponents of LIH means that there are "central or conceptual modules as well as peripheral input and output modules, but […] the mind also contains some significant non-modular systems or processes." (CARRUTHERS 2003a, 67) (cf. CAREY & SPELKE 1994). For the purpose of the current analysis, the most important proposition is that the linguistic module serves as the integrator of output information from other modules, allowing for conscious formation of beliefs and making decisions (CARRUTHERS 2002; 2003a).

Some other central points need to be made at this point. The consequence of the integrative function of language conceived in the above way is that it neatly harmonizes the non-modular central system with the modular systems. This allows us to accept the thesis of the non-modularity of the central system without necessarily assuming its clear-cut separation from the modular architecture of the rest of the mind. The nature of the language faculty is such that it is closely linked to the information stemming from the modules. It does not take all the information blindly or in random sequence, however. Rather, it intelligently chooses specific bits of information and arranges them into novel units (CARRUTHERS 2003a). The main problem of the proposition will be thus to identify the aspects of natural language that fulfill that function. The main candidates here are syntax with its recursive and abstract properties, semantic compositionality, and certain phonetic properties.

To support the thesis of the integrative function of language three points are usually made. First, proponents of the view claim that we have strong reasons to believe that linguistic faculty is a module with a dedicated input and output, and a proprietary database. Second, there are strong reasons to believe that the language faculty is located in the architecture of the mind in a way that allows its integrative function — to take in information from the modules and to produce it in the form of speech. Third, and most importantly for us, the abstractness and combinatorial structure of syntax supply the tools for taking information from the modules and combining them into coherent units of linguistic format.

The last point is articulated in three more specific points. First, any linguistic performance is on this view always undergirt by a mechanism that instantiates some generative rule, rather than a simple reflex or habitual behavior (cf. CHOMSKY 1972/2006). Second, the creative property of language stems mostly from its syntactic connectives, such as the functor "or" or "and," or pronouns, which allows the combination of two sentences into one recursively.[1] Third, the input information is said to be coded in the linguistic format and stored that way, which allows it to be reproduced and re-combined flexibly, and articulated in speech at hand. The new format imposes its specific limits onto the data, however, and any recombination and articulation must conform to the syntactic rules of the linguistic format, which determines the specific form of the human thought. This formal constraint will have its effect on the shaping of the content acquired from the non-linguistic modules; we may get information from non-linguistic sources, but the way we present it to ourselves will be essentially linguistic, and so the process of thinking will abide by the rules of linguistic production.

The above view does justice to the undeniable fact of flexibility of human thought within the framework of modularism. Humans create new concepts and break up old ones with ease; think of the child's creativity in pretense or fantasizing, or metaphorical and hypothetical thinking. The creativity in question is characterized by its independence from any specific context or sensory stimulation, and as such has always been held up as an argument against modularism, especially its broad versions. LIH offers a way out of that criticism without sacrificing much of modularism.

Language is thus to be the reason for the flexibility of human thought at the level of the central system. A welcome consequence of the view is that

[1] Generally, the rationale here is as follows: we are able to use predicates from one domain in another. For instance, if the predicate G(x) is native to one domain, we need to "detach" it from its usual objects and reapply it to objects from another domain. To do that, we need to be able to grasp what it means to be predicated with G, to comprehend the quality as separate from its instantiations. Proponents of LIH claim that one needs a special cognitive system that will achieve such a task. And language with its syntactic features is an obvious candidate: "The relative clause allows complex predicates to be extracted from complete thought in a form that will allow them to be applied to objects falling under other cognitive domains, and the defining feature of domain-general cognition is that objects from one domain can be thought about in terms formerly associated only with objects from another domain, as in totemic art when an artifact is attributed the properties of an animal, or as in advanced tool construction when the design of a tool is specifically tailored to properties of the intended prey." (BERMUDEZ 2002, 667). Proponents of LIH claim that the relative clause allows to think of predication as seperate from its object.

since the format of thought is linguistic, frameworks and analyses coming from research on natural language can inform the cognitive questions.

Summing up, according to LIH, language is said to function as the faculty that allows integration of domain-specific mental representation and creation of flexible thought. The properties by virtue of which language does so are claimed to be syntactic (combinatoriality) and semantic (compositionality) (BERMUDEZ 2002; CARRUTHERS 2003b, 511–512; GENTNER & GOLDIN, 2003). Empirical interest of the framework falls on those cognitive processes that can be most reasonably seen as integrating information from different modules. In our exposition we will discuss two of them: spatial thinking and mathematical thinking.

3. LANGUAGE AND TRANSCENDING THE LIMITS OF SPATIAL THINKING

The first case of LIH research is the attempt to show that language integrates geometric information about the location of the object with the information about the object's properties such as color or size (HERMER-VAZQUEZ et al. 1999). This means that it first has to be demonstrated that without language the information from the modules is not integrated and is tapped exclusively disjunctively during the subject's activity. Then, it needs to be shown that linguistic mastery allows for the integration of the data from the two modules. That methodology aims at showing that it is language that enables the subject to transcend the computational limitations of the two basic modules.

Children before the age of two are first demonstrated to represent the geometrical structure of the environment. In rectangular rooms, they distinguish the corners with the short wall on the left from the corners with the short wall on the right. At around the same time, children are shown to be able to represent non-geometric properties of the environment.[2]

Hermer-Vazguez and Spelke have repeatedly shown that children before the age of three do not use both of the above kinds of representation, even

[2] Importantly, these skills are not connected in any way with previous linguistic experience. Linguistic forms that express the information coming from these representations are said to mirror their structure in the form of concepts or thoughts. It is thus assumed that certain kinds of concepts are prior to language and form the basis for the acquisition for corresponding linguistic concepts. These prior concepts are independent of language. (See HERMER-VAZQUEZ et al. 1999, 9).

though they are capable of doing so separately. More specifically, looking for an object in a rectangular room children before four (as well as animals) rely only on geometrical information, not on information of other kinds — like color — that is still available in the task.

The interpretation of the above is that two separate systems have developed in phylogeny: one for representing geometrical properties such as "in the corner with the long wall on the right and with the short wall on the left"; and one for representing properties of objects such as shape or color. In research from 1994 that adapted previous methods for studying spatial orientation in rats (CHENG 1986), Hermer and Spelke demonstrated that children younger than three use only geometrical information to retrieve an object in a rectangular room. The room had identical containers in each of its corners. An attractive object was hidden in one of them with the child observing the process. Then, the child's eyes were covered and she was turned around a few times to disorient her, after which her eyes were uncovered and she was instructed to go and find the object. The results were equally distributed between the two opposite corners that were identical in only geometrical features — long wall on the right, short wall on the left. Children did look in both of these locations even when one of the walls in the room was of a different color, which objectively broke the symmetry and did away with all the ambiguity about the location of the object. This means that the children did not take into consideration that piece of information as they systematically behaved as if only geometrical information was available. Further experiments strengthened the conclusion by showing that children act the same even when the extra non-geometrical information is provided with objects known to them. When the object was hidden in two perceptually different containers, but which were still in the two opposite corners, children still searched in both of them equally often. However, when the two containers were put in the center of the room, children were able to choose correctly on the basis of the container's properties, which shows that they were able to represent that information when geometrical structure of the room was not being considered.

The above methodology has become the model for research on spatial orientation (see a review in CHENG, HUTTENLOCHER, & NEWCOMBE 2013 and SPELKE & LEE 2012). Spelke and Lee (2012) argue that egocentric distance from objects and directions in a limited space is the basis for reorientation in disorientation. That hypothesis has been supported by neurophysiological research that showed that in the hippocampal areas of the brain there are neurons selectively reactive to those two properties of space. The same activity pattern has been found in neuroimaging studies with people who were performing a task that required orientation in space. Still, there are other orientation systems already present even in small children. In natural conditions, complete disorientation happens rarely. Usually, some information about geometrical features of the environment is constantly available, which makes possible to focus on object properties that are represented by the second system of spatial representation.

The second system deals with representing the shapes of objects. In contrast to the first system, it represents individual objects but relies on simple measures of angle and relative length to identify them (for more details on the characteristics of the system see SPELKE & LEE 2012, 2789). It does not consider orientation and distance (in relation to the subject). As such, it, as it were, complements the first system. Spelke and Lee claim that this has an adaptionist story. To navigate in space, one needs to represent egocentric distance and directions. On the other hand, classifying objects is also important and to that end one needs to consider properties relative to the objects themselves, which allows for representing their identity regardless of the subject's perspective.

LIH proponents take the above findings to mean that there are two separate domain-specific modules housing two distinct kinds of information: that "something is located in the corner with the long wall on the left" and that "something is located close to the blue wall." The research cited is said to show that these two types of information are not integrated at first. Young children rely solely on geometrical information and it seems that they cannot integrate that information with representation of non-geometrical properties of objects while dealing with the task in the test. On the other hand, older children and adults do integrate the two kinds of information with ease. Importantly, the studies showed no significant correlation between performance and age, non-verbal IQ, verbal short memory, vocabulary size, and understanding spatial terms. In keeping with LIH, children keep searching in both corners despite the colored wall breaking up the symmetry of the room until linguistic competence allows creating complex representations of the type "on the left, next to the blue wall." The ability to create such representations, which mix information from the two modules, coincides with the acquisition of some capacities to create new types of sentences in natural language.

Another relevant series of experiments were the dual task studies. The idea was that the performance of actions that required incorporating information from two different modules would worsen if the person was at the same time engaged linguistically. In one of the tasks, the participants were asked to perform a spatial orientation task while at the same time repeating speech they were listening to on headphones. In another one, they were asked to do the same task, but had to tap along the rhythm played on the headphones. Their hypothesis had it that following the speech would occupy the linguistic faculty while following the rhythm would not.

> The results of these experiments were striking. Shadowing of speech severely disrupted subjects' capacity to solve tasks requiring integration of geometric with non-geometric properties. In contrast, shadowing of rhythm disrupted subjects' performance relatively little. Moreover, a following experiment demonstrated that shadowing of speech didn't disrupt subjects' capacities to utilize non-geometric information per se—they were easily able to solve tasks requiring only memory for object-properties. So it would appear that it is language itself which enables subjects to conjoin geometric with non-geometric properties, just as the hypothesis that language is the medium of cross-modular thinking predicts. (CARRUTHERS 2002, 11).

The results obtained are said to support LIH:

> Although the compositional semantics of a natural language intricate and not fully understood, one thing is clear: the rules of combining words in a sentence apply irrespective of the core knowledge system that constructs the representation to which each word refers. Once a speaker has learned the expression *left of X* and a set of terms for people, places, numbers, events, objects, collections, emotions, and other entities, she can replace X with expression that refer to entities from any and all of those domains (e.g., left of the house where the happy old man cooked a 14-pound turkey for his family last Thanksgiving) natural language therefore can serve as a medium for forming representations that transcend the limits of domain-specific, core knowledge system. (SPELKE 2003, 296).

The above hypothesis forces us to adopt an unorthodox view on speech production. Traditionally, speech starts from thoughts; it begins with non-linguistic mental representation and then the subject uses linguistic means (syntax, lexis, phonetics etc.) to express that thought (cf. LEVELT 1989). Clearly, one cannot accept such a view in light of the current perspective. Naturally, the thought "the toy is on the right hand side from the blue wall" cannot be conceived of before linguistic formulation if it is language that makes it possible to be created in the first place.

Proponents of LIH stress that the above is not the only support for their hypothesis. Research on mathematical cognition is another empirical backing of LIH. In that context as well, researchers point to language as the source of specifically human kinds of thought (CARRUTHERS 2002; LEE 2017; SPELKE 2003; SPELKE & TSIVKIN 2001). It is of course only the simplest aspects of mathematical cognition that are considered here, such as numerosity distinction, counting etc. It is understood, however, that these basic skills lead the way to other, more refined ones. Numerical representation is a taken for granted aspect of everyday life and we may often forget how special humans are in this respect. Hence, it is understandable that the proponents of LIH have turned to the development of mathematical skills to test their hypothesis.

4. TWO SYSTEMS OF MATHEMATICAL THINKING

There are five basic reasons why mathematical thinking is under scrutiny by proponents of LIH: First, there is a deep-seated conviction that mathematical skills are the specifically human capacity par excellence. Second, it is assumed that by demonstrating how humans use the concept of number we can clearly show how different we are from other animals. Third, mathematical thinking seems to be a great example of the kind of capacity that integrates a lot of diverse skills, such as categorization, or differentiating categories on the basis of numerosity. Four, there are grounds to believe that the ability to understand that adding one to any set will give us a set one-element bigger is made possible by memorizing linguistic sequences that refer to counting. Five, mathematical thinking is assumed to be the domain where differences between nominalism and different kinds of realism are foregrounded, which allows a wider discussion.

Mathematical prowess distinguishes humans from other animals. The arithmetic skills such as unconstrained counting, using fractions, or using the concept of zero or negative numbers radically separate us from other species. Rudimentary mathematical skills of some animals notwithstanding, there is an obvious chasm between humans and other beings (BARROW 1992; CARRUTHERS 2012; GRIFFIN 1992; SPELKE & TSIVKIN 2001, 46; WYNN 1992b).

The abstract concept of number has been seen as specifically human for a long time. The ability for symbolic representation has been argued to be the crucial source of human mastery of the number. The oldest records of symbolic representation of the number go back 20,000 years to the time when humans used tokens to help themselves count. A proper symbolic representation of the number was created around 4000 BCE. It is not clear when numerals entered language, but even today there are languages that do not have words for numbers above 2, 3 or 4. Nevertheless, there is much evidence coming from cross-cultural studies that shows the benefits that a proper system of digits affords, and the limits that an improper one imposes (Barrow 1992).

Neuroimaging studies support LIH as they demonstrate that during performing calculations the same brain areas are active that take part in linguistic processing (Feigenson, Dehaene, & Spelke, 2004, 307–314; Gelman & Butterworth 2005, 6–7; Laurence & Margolis 2008). Contemporary research demonstrates that the ability to represent numbers is present in the first months of life in humans and other vertebrates. It is separate from basic perceptual mechanisms and abstract (that is, independent from particular objects being counted and perceptual modality). For instance, newborns can match sets of a dozen stationary objects with sequences of the equivalent number of sounds (Izard et al. 2009). Naturally, they do not do so exactly, but approximately, which does not change the fact that the property tapped is an abstract concept of number that crosses sensory modalities as well as modes of presentation (simultaneous and sequential).

In the literature, we find two basic mathematical systems: amodal, analog system of the approximate numerosity that represents high numbers, and the exact numerical system that represents small numbers. These two systems provide the evolutionary basis for the mathematical skills present in many species (Feigenson et al. 2004). They are developmental fundamentals that develop irrespective of education or other ontogenetic variables; neither do they change once they have achieved maturity in childhood, and function throughout life once they have done so. Proponents of LIH will naturally see language as integrating the two core systems and amplifying human capacities way beyond that which the two of them afford separately. Taken separately, the two systems are highly limited: "Neither system supports concepts of fractions, square roots, negative numbers, or even exact integers" (Feigenson et al. 2004, 307).

The first system is specialized in processing information about numerosity regardless of whether it is in a set or a sequence. Empirical data show that it activates automatically and independently from sensory modality. It allows estimating and comparing sets' sizes, and works according to Weber-

Fechner's psychophysical law, which describes how it is easier to distinguish two stimuli depending on their intensity. For numerical distinctions, it means that the relative difference between the two sets determines the ability to assess which one of them is bigger than the other. For example, a four-year-old can pick an eight-element set as the bigger one when the other has six elements. The same absolute difference — two — is however not enough to allow the same child to correctly pick the bigger set from the sets of nine and eleven elements. On the other hand, it poses no problems to make a correct choice when the two sets have nine and twelve elements as the relative difference falls within the necessary ratio — three to four — that is, the same as in the case of six- and eight-element set pair. The precision of this phenomenon is described with Weber's fraction, which is calculated as the ratio of the smallest perceivable difference to the weaker one of the stimuli; here, it is the ratio of the difference between the two numbers to the smaller one of the two numbers.

Two conclusions follow from Weber-Fechner's law: the effect of size and the effect of distance. The former means that processing high numerosities requires more computational resources (and effectively more time). The latter means that distinguishing between numbers relatively close (their difference closer to Weber's fraction) requires more resources than distant numbers. These two properties suggest that numbers may be represented in a quasi-spatial, analog way: the value of subsequent numbers may be represented as a positon on a one-dimensional scale, with a starting point and a vector ordering the elements from the lowest to the highest, as well as a logarithmic measure of distance between the elements determined by Weber's fraction. Such a representation of the number will differ considerably from the concept of number we use every day. Inasmuch as we too standardly represent numbers spatially, we see them as points and the distances between them as constant, while in the analog system, the number's position is fuzzy, and the distance between particular numbers is relational rather than absolute.

The second system does not have such limitations; it represents numbers precisely. Its limits consist in how many objects it can actually count — around 3 and 4. Number representation of this system is as it were a side effect of its primary function to pay attention to and track a couple of objects at the same time. The system needs to represent individual object identity in spite of its changing position (movement), and occasional disappearance behind obstacles (getting out of the visual field). For that purpose, it seems, the system needs to code the number of the tracked objects.

The effectiveness of this system is nicely illustrated by a simple experiment. In good conditions and without any distractions, we are capable of picking out from a collection of squares the four ones that we decided to track a minute ago. It is not clear, however, whether the system is exclusive to the visual modality or if it is amodal like the first system.

The functioning of the two systems, their evolutionary roots, and mutual independence are demonstrated by a recent series of experiments on fish — Endler's guppies, among others. Guppies, like the social species they are, increase their chances of survival by joining the bigger school. Their ability to distinguish numbers can be therefore measured by observing which schools they join if they have a choice between two that have different number of fish in them, which can be manipulated by changing the number in each school. In the experiment, there was an aquarium separated with glass walls into three compartments. One fish was put into the middle compartment while the other two compartments were occupied by two schools of different sizes. If the schools were not bigger than four individuals, the middle guppy always spent more time by the wall of the compartment where the bigger school was. However, if the two schools had more than four individuals, the guppy chose the bigger school correctly only if the ratio of the number of one school to the other did not exceed 1 to 2. Additionally, the preference decreased as the size of the schools increased, which agrees with the effect of distance. The conclusion is that the former case demonstrates the functioning of the precise system, while the latter evidences the approximate system.

Interestingly, when the size of one of the schools was lower than four and the other was higher than four, the fish preferred the bigger group only when the ratio reached one to two (e.g. 3 to 6, but not 3 to 5). However, there was not an effect of distance; the preference did not fall together with a decrease in the ratio. Similar discontinuity was found in infants (Feigenson & Carey, 2005; van MARLE & WYNN 2011): Twelve-month-olds chose the box where more biscuits were put (1 vs. 2, 2 vs. 3) as long as none of the two boxes contained more than three biscuits. When there were more than three biscuits in one of the two boxes, children chose at random, even when the difference was quite big (e.g. 1 vs. 4 or 3 vs. 6). When, however, both boxes contained a number of biscuits that fell within the range of high numbers (e.g. 5 and 10), the children went back to choosing the box with the higher number of biscuits (cf. CORDES & BRANNON 2009). This phenomenon has been established for other species (cf. ANDERSON & CORDES 2013).

To wit, the results show that: (1) both of the two systems are present in animals and have the same characteristics as in humans; (2) the two systems are independent from each other and work automatically depending on the size of the calculated sets; (3) the systems provide abstract information in the sense that it is not bound to any particular action; sometimes it is used for social life (choosing a group of conspecifics) and sometimes, for other species, it serves different purposes. However, it seems impossible for both human and non-human subjects to integrate information coming from the two systems. The two systems are thus domain-specific, relatively hermetic, and their output is not computed together with information coming from other systems.

In summary, we may say that we have two kinds of representation of the number: the first one pertains more to the object itself, the other more to the numerosity.

> One system represents small numbers of persisting, numerically distinct individuals exactly and takes account of the operation of adding or removing one individual from the scene. It fails to represent the individuals as a set, however, and therefore does not permit infants to discriminate between different sets of individuals with respect to their cardinal values. A second system represents large numbers of objects or events as sets with cardinal values, and it allows for numerical comparison across sets. This system, however, fails to represent sets exactly, it fails to represent the members of these sets as persisting, numerically distinct individuals, and therefore it fails to capture the numerical operations of adding or subtracting one. Infants therefore represent both "individuals" and "sets," but they fail to combine these representations into representations of "sets of individuals." (SPELKE 2003, 299).

It needs to be added that the two systems are claimed to be separate and not to share any representational resources. This clear-cut delineation is central to the hypothesis. First, the high number system is sensitive to relation of numerosities based on approximate evaluation, it is relative; the small number system reacts to the absolute number of individual elements. Second, the high number system is indifferent to incremental changes where one element is added to the set at a time, whereas the small number system is sensitive to such changes (FEIGENSON et al. 2004, 311). Third, small number representation is limited to 3 for children and 4 for adults. The high number system is not limited in that sense (SPELKE 2003, 297; XU 2003, B19, B23). Four, the high number relational representation is limited by the range of Weber's fraction: 1.5-2 for six-month-olds, 1.2-1.5 for nine-month-olds, and around 1.15 for adults. The small number system does not have such limitations; the child can distinguish 2 from 3 even though the Weber fraction is below the threshold in this case. Five, for the high number system the size and arrangement of the elements of the sets does not make a difference. The small number representation is sensitive to such changes.

The above review should make it clear that the basic numerical cognition is highly limited. What seems to be missing is the concept of a set of individuals, which would allow for representing the exact number of elements of a set without any limits on its size. When such a concept is missing, numerical expressions represent either arrangements of single objects or approximate numerosities. Without it, the organism cannot represent the cardinal number of the elements in the set in the way that allows for adding or subtracting one element to change the representation of that set. What is more, a system without such a concept is not generative — it does not have the rule to create next higher natural numbers, and it is unable to represent discrete infinity (FEIGENSON et al. 2004, 311). It is the postulate of LIH that creating this concept is effected by combining the information of the two systems through language (SPELKE 2003, 301).

5. LANGUAGE'S ROLE IN MATHEMATICAL COGNITION

Spelke and Tsivkin conducted three experiments that studied the role of a given language in people's numerical representation. They came out of the observation that people speaking two or more languages tend to count and do arithmetic only in one of their languages, mostly in the language in which they learned mathematics. For example, a person who has been speaking a second language almost exclusively for a long time still uses her first language to count and do math (SPELKE & TSIVKIN 2001, 47).

In the studies, bilingual students were taught new numerical operations, new mathematical equations, and new historical and geographical facts that contained both numerical and non-numerical information. The material was taught interchangeably in their two languages. They were tested in two of the languages always. The studies revealed a pattern that students were more effective in tests on material containing numerical elements if the test was in the same language as the language the material was taught. Approximate numerosities and non-numerical facts did not produce that effect. Dehaene, Spelke, Pinel, Stanescu, and Tsivkin (1999) provide consistent data: In their study, they specifically targeted the approximate-discrete distinction and demonstrated that language impacts calculating only when discrete values are involved, not approximate ones.

These results are interpreted as support for LIH. Numerical representation is linguistically constituted, and therefore being tested in a different language is more computationally costly to solve the problems. Various studies have shown that counting is the last thing we master in a foreign language and that bilingual people do calculations in the language that was their language of instruction in mathematics, and some multilinguals concede that counting is the domain in which it is hardest to switch codes (BIALYSTOK 1999; GROSJEAN 2001; MARZECOVÁ et al. 2013; WODNIECKA et al. 2018).

Another significant support to LIH comes from neuroimaging studies. Some brain regions in the intraparietal sulcus and prefrontal cortex are activated when processing linguistic numbers and estimating the sizes of sets. What is more, new methods of analysis in fMRI studies have shown that neuronal populations in these regions react the strongest to a particular numerical value, and part of them reacts only to a given modality (sets, words, digits) and part is common to all of modalities. These "numerical neurons" display scalar properties that are characteristic to the high number system: The range of numerical value to which a given neuron reacts and the variance of its firing increase together with Weber-Fechner's law. However, the variance is smaller for symbolically represented values.

Another piece of data comes from recent research that compared the precision of the high number system of a person, described with Weber's fraction and their academic performance at math. Both longitudinal and cross-sectional studies done with children and adults have confirmed a positive relationship between the two. The more effective the high number system (Weber's fraction lower) the better performance at math (measured with either grades or standard tests) (DEHAENE 1999; NIEDER & DEHAENE 2009).

A yet another study that supports the thesis comes from developmental research. Lipton and Spelke (2005) showed that five-year-olds can give an approximate number of a set's elements within the range that they can count (the study focused on the range of 20–120). That is, the issue here is not counting the elements (that ability is naturally linguistic) but the ability to quickly estimate the size of the set without counting. The study found that the distribution of the estimation mapped onto the predictions done with the high number system.

As far as the small number system is concerned, the question of its relation to later conceptual knowledge of numbers is less clear. Neuroimaging have shown an activation of two different regions; however, both of them were in the intraparietal sulcus and neighboring areas. However, Carey and colleagues (CAREY 2009; LE CORRE & CAREY 2007) argue that the small number system can be necessary for the acquisition of the linguistic concept of number. In Le Corre and Carey's model, the child uses the grammar of the language to distinguish between the number *one* and *more than one* (basing on plural and singular distinction in grammar). Usually, it happens in the first months of the third year of life, but it also depends on the particular language's grammar, whether it has a clear singular-plural distinction or not (some far Eastern languages do not have that distinction and the skill emerges around half a year later (SARNECKA et al. 2007). Then, in few-month-long intervals, up to the age of around four, the numbers from the range 2-4 are mastered. They are, however, represented not as abstract numbers, but rather as models of objects arranged in space (see JOHNSON-LAIRD 1980).

Karen Wynn (1992a) showed that the ability to count elements of a set beyond the number four is limited only by one's knowledge of the list of numerals. However, children can count up to twenty before they actually comprehend the concept of number, that is map them onto the high number system. In Le Corre and Carey's (2007) studies, it was not until six months after being able to list numerals that children had evinced the scalar way of thinking about numbers that is characteristic of the high number system (the effects of distance and size).

Fresh insight into the issue of language's role in numerical cognition is provided by two studies on two subsistence societies (GORDON 2004; PICA et al. 2004). The Amazonian tribes of Piraha and Mundurucu have really limited numerals in their languages. The former one has words only for "one" and "two," while the latter additionally for "three," "four," and "five." Both of them, however, use the numerals in an approximate way; admittedly, "two" is most often used to refer to two objects, but sometimes to three or one as well. The studies demonstrated that people from these societies can only compare sets approximately, just like Western children who cannot count yet (DEHAENE 1999; NIEDER & DEHAENE 2009). It follows then that their concept of number is entirely reliant on the high, approximate, or analog number system.

6. SUMMARY AND DISCUSSION

In the final section, we take up four issues present in the context of the above research. Specifically, we discuss mathematical cognition and offer some criticism of the generalizations about language's role that tend to be made in the literature.

(1) It is questionable whether the small number system is a numerical system at all. The ability to distinguish one from two relies on representing one object and then "one and another object" (XU & SPELKE 2000, B3). This mechanism has been argued to be an object-perception mechanism rather than a number-perception one. Indeed, infants are said to "represent objects but not sets with cardinal values" (XU & SPELKE 2000, B3). Moreover, Gelman and Butterworth claim that we should speak of an object-file mechanism in this case, the essence of which consists in paying attention to two or three objects rather than the property of two-ness or three-ness (GELMAN & BUTTERWORTH 2005, 6). Similarly, the high number system is open to the same critique: The ability to distinguish between sets of 8 and 16 may be a mechanism that picks out only certain regularities in the arrangement of objects. Granted that certain perceptual properties of the set — like spatial distribution, color, or the kind of the objects — do not affect the workings of the system, it is still a fact that the ability relies solely on perceptual data. Both systems are implicated perceptually; the only information considered is one coming from the senses (DUMMET 1994, 121–126). On that view, linguistic numerals exist alongside these abilities and function only as adjectives describing a particular state of things (cf. BARROW 1992; DAVIDSON 1984/2001). The two systems from this perspective are a case of pictorial thinking. The linguistic integration of them would not provide so much benefit if there was not a significant augmentation of the systems effected by the merge.

Both of the abilities are reliant on and limited by perceptual data. On the other hand, mature arithmetic skills are free from such limits. The sheer combination of the two systems without any modifications to them could not generate the starkly different system that we get in effect. It seems then that we are left with two interpretative moves: language's influence is either wider or we have to consider additional, non-linguistic factors in the development of the mature ability. The former is inconsistent with the assumptions on which the discussed modular conception of the mind rests; the latter is inconsistent with the hypothesis that uniquely human thought depends on language.

(2) Spelke and Tsivkin consider the exact number representation — an actual numerical representation — as coming from the small number system

(cf. SPELKE & TSIVKIN 2001, 82–83). In the light of the above point, this seems to be an attempt to overinterpret the data about the small number system in order to refute the anti-numerical arguments discussed. As Margolis and Laurence righty point out, when we assume that "the small-number system represents a two-member set as "'an object x and an object y, such that y ≠ x,' whereas the large-number system represents it as 'a blur on the number indicating a very small set'" (SPELKE & TSIVKIN 2001, 85) then a coherent combination of the two kinds of representation becomes problematic (LAURENCE & MARGOLIS 2005, 230).

(3) In the context of the present discussion, we need to note the data that suggest (a) that different regions of the brain are responsible for language and the number, and that (b) linguistic deficits do not necessarily lead to corresponding deficits in mathematical skills that purportedly stem from the role of language in augmenting the function of the two basic systems (cf. CIPOLOTTI & VAN HARSKAMP 2001; GELMAN & BUTTERWORTH 2005).

(4) Lila R. Gleitman and Anna Papafragou (2005) in their discussion of Spelke and Carruther's views rightly point out that granting the truth of the LIH brings out the question of the differences of mathematical concepts across different languages. Even in English, recursive properties are not obvious:

> Specifically, until number eleven, the English counting system presents no evidence of regularity, much less of generativity: a child hearing one, two, three, four, five, six up to eleven would have no reason to assume — based on properties of form — that the corresponding numbers are lawfully related (namely, that they successively increase by one). For larger numbers, the system is more regular, even though not fully recursive due to the presence of several idiosyncratic features (e.g., one can say eighteen or nineteen but not tenteen for twenty). In sum, it is not so clear how the 'productive syntactic and morphological structures available in the counting system' will provide systematic examples of discrete infinity that can then be imported into number cognition). (GLEITMAN & PAPAFRAGOU, 2005, 655).

It appears then that the LIH implies certain assumptions about the structure of natural languages and ways of their acquisition. This is provided by the Chomskian model of language acquisition where cognitive structures responsible for language are innate and allow for its generative properties (cf. CHOMSKY 1972/2006). Understood this way, it is easy to see how the same generative processes taking part in language functioning and acquisition could help unite information from the two mathematical modules.

REFERENCES

Anderson, Ursula, & Sara Cordes. 2013. "1 < 2 and 2 < 3: non-linguistic appreciations of numerical order." *Frontiers in Psychology* vol. 4, 5. DOI: https://doi.org/10.3389/fpsyg.2013.00005.

Baddeley, Alan D. 1986. *Working memory*. Oxford: Clarendon Press.

Barrow, John D. 1992. *Pi in the sky: Counting, thinking, and being*. Oxford: Clarendon Press.

Bermudez, José Luis. 2002. "Domain-generality and the relative pronoun." *Behavioral and Brain Sciences* vol. 25, issue 6: 676–77. DOI: https://doi.org/10.1017/S0140525X02240121.

Bermúdez, José Luis. 2003. *Thinking without words*. (Philosophy of mind series). New York, Oxford: Oxford University Press.

Bialystok, Ellen. 1999. "Cognitive Complexity and Attentional Control in the Bilingual Mind." *Child Development* vol. 70, No. 3: 636–44. DOI: https://doi.org/10.1111/1467-8624.00046.

Brannon, Elizabeth M. 2002. "The development of ordinal numerical knowledge in infancy." *Cognition* vol. 83, issue 3: 223–40. DOI: https://doi.org/10.1016/S0010-0277(02)00005-7.

Carey, Susan. 2009. *The Origin of Concepts*. Oxford: Oxford University Press.

Carey, Susan, & Elizabeth S. Spelke. 1994. "Domain-specific knowledge and conceptual change." In Lawrence A. Hirschfeld & Susan A. Gelman (eds.). *Mapping the Mind: Domain Specificity in Cognition and Culture*, 169–200. Cambridge: Cambridge University Press.

Carruthers, Peter. 1996. *Language, thought and consciousness: An essay in philosophical psychology*. Cambridge, England, New York, N.Y: Cambridge University Press. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db= nlabk &AN=54512

Carruthers, Peter. 2002. "The cognitive functions of language." *Behavioral and Brain Sciences*, vol. 25, issue 6: 657–74. DOI: https://doi.org/10.1017/S0140525X02000122.

Carruthers, Peter. 2003a. "Moderately massive modularity." In *Royal Institute of Philosophy supplement, 1358-2461*. Vol. 53: *Minds and persons*, edited by Anthony O'Hear, 67–90. Cambridge: Cambridge University Press.

Carruthers, Peter. 2003b. "On Fodor's Problem." *Mind and Language* vol. 18, No. 5: 502–23. DOI: https://doi.org/10.1111/1468-0017.00240.

Carruthers, Peter. 2012. "Language in cognition." In Eric Margolis, Richard Samuels, & Stephen P. Stich (eds.). *Oxford handbooks online. The Oxford handbook of philosophy of cognitive science*, 382–401. New York: Oxford University Press. DOI: https://doi.org/ 10.1093/ oxfordhb/9780195309799.013.0016.

Cheng, Ken. 1986. "A purely geometric module in the rat's spatial representation." *Cognition* vol. 23, issue 2: 149–78. DOI: https://doi.org/10.1016/0010-0277(86)90041-7.

Cheng, Ken, Janellen Huttenlocher, & Nora S. Newcombe. 2013. "25 years of research on the use of geometry in spatial reorientation: a current theoretical perspective." *Psychonomic Bulletin & Review* vol. 20, issue 6: 1033–54. DOI: https://doi.org/10.3758/s13423-013-0416-1,

Chomsky, Noam. 1972/2006. *Language and mind.* 3rd ed. Cambridge: Cambridge University Press.

Cipolotti, Lisa, & N.J. van Harskamp. 2001. "Disturbances of number processing and calculation." In *Handbook of neuropsychology*. Vol. 3. *Language and aphasia*, edited by R[ita] S[loan] Berndt. 2nd ed., 305–44. Amsterdam, London: Elsevier.

Cordes, Sara, & Elizabeth M. Brannon. 2009. "The relative salience of discrete and continuous quantity in young infants." *Developmental Science* vol. 12, No. 3: 453–63. DOI: https://doi.org/10.1111/j.1467-7687.2008.00781.x.

DAVIDSON, Donald. 1984/2001. "Thought and Talk." In Donald DAVIDSON. *Philosophical essays*. Vol. 2. *Inquiries into truth and interpretation.* 2nd ed., 155–70. Oxford: Clarendon Press. DOI: https://doi.org/10.1093/0199246297.003.0011.

DEHAENE, Stanislas. 1999. *The number sense: How the mind creates mathematics* (Oxford Univ. Press pbk. ed.). New York: Oxford University Press.

DEHAENE, S[tanislas], E[lizabeth] SPELKE, P. PINEL, R. STANESCU, & Sanna TSIVKIN. 1999). "Sources of mathematical thinking: behavioral and brain-imaging evidence." *Science* vol. 284 (5416): 970–74.

DUMMET, Michael. 1994. *Origins of Analytic Philosophy*. Cambridge, Mass.: Harvard University Press.

FEIGENSON, Lisa, & Susan CAREY. (2005). "On the limits of infants' quantification of small object arrays." *Cognition* vol. 97, issue 3: 295–313. DOI: https://doi.org/10.1016/j.cognition.2004.09.010.

FEIGENSON, Lisa, Stanislas DEHAENE, & Elizabet S. SPELKE. 2004. "Core systems of number." *Trends in Cognitive Sciences* vol. 8, issue 7: 307–14. DOI: https://doi.org/10.1016/j.tics.2004.05.002.

GELMAN, Rochel, & Brian BUTTERWORTH. 2005. "Number and language: how are they related?" *Trends in Cognitive Sciences* vol. 9, issue 1: 6–10. DOI: https://doi.org/10.1016/j.tics.2004.11.004.

GENTNER, Dedre, & Susan GOLDIN-MEADOW (eds.). 2003. *A Bradford book. Language in mind: Advances in the study of language and thought*. Cambridge, Mass., London: MIT Press.

GLEITMAN, Lila R., & Anna PAPAFRAGOU. 2005. "New perspectives on language and thought." In Keith J. HOLYOAK & Robert G. MORRISON (eds.). *The Cambridge handbook of thinking and reasoning.* Cambridge 543–568. Cambridge, New York: Cambridge University Press.

GORDON, Peter. 2004. Numerical cognition without words: evidence from Amazonia. *Science* vol. 306 (5695), 496–99. DOI: https://doi.org/10.1126/science.1094492.

GRIFFIN, Donald R. 1992. *Animal minds*. Chicago: University of Chicago Press.

GROSJEAN, François. 2001. *Life with two languages: An introduction to bilingualism*. Cambridge, Mass.: Harvard University Press.

GUMPERZ, John J., & Stephen C. LEVINSON. 1996. *Rethinking linguistic relativity*. *Studies n the social and cultural foundations of language*. Vol. 17. Cambridge: Cambridge University Press.

HERMER, Linda, & Elizabeth S. SPELKE. 1996. "Modularity and development: the case of spatial reorientation." *Cognition* vol. 61, issue 3: 195–232.

HERMER-VAZQUEZ, Linda, Elizabeth S. SPELKE, & Alla S. KATSNELSON. 1999. "Sources of flexibility in human cognition: dual-task studies of space and language." *Cognitive Psychology* vol. 39, issue 1: 3–36. DOI: https://doi.org/10.1006/cogp.1998.0713.

HESPOS, Susan J., & Elizabeth S. SPELKE. 2004. "Conceptual precursors to language." *Nature* vol. 430 (6998): 453–456. DOI: https://doi.org/10.1038/nature02634.

IZARD, Véronique, Coralie SANN, Elizabeth S. SPELKE, & Arlette Streri. 2009. "Newborn infants perceive abstract numbers." *Proceedings of the National Academy of Sciences of the United States of America* vol. 106 (25): 10382–85. DOI: https://doi.org/10.1073/pnas.0812142106

JOHNSON-LAIRD, P[hilip] N. 1980. "Mental Models in Cognitive Science." *Cognitive Science* vol. 4, issue 1: 71–115. DOI: https://doi.org/10.1207/s15516709cog0401_4.

KURCZ, Ida. 1976. *Psycholingwistyka. Przegląd problemów badawczych*. Warszawa: Państwowe. Wydawnictwo Naukowe.

LAURENCE, Stephen, & Eric MARGOLIS. 2005. "Number and Natural Language." In Peter CARRUTHERS, Stephen LAURENCE, & Stephen STICH (eds.). *Evolution and Cognition. The* *innate mind.* Vol. 1. *Structure and Contents*, 216–35. New York, Oxford: Oxford University Press.

LAURENCE, Stephen, & Eric MARGOLIS. 2008. "Linguistic determinism and the innate basis of number." In Peter CARRUTHERS, Stephen LAURENCE, & Stephen STICH (eds.). *Evolution and Cognition. The innate mind.* Vol. 3. *Foundations and the Future*, 139–61. New York, Oxford: Oxford University Press.

LE CORRE, Mathieu, & Susan CAREY. 2007. "One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles." *Cognition* vol. 105, issue 2: 395–438. DOI: https://doi.org/10.1016/j.cognition.2006.10.005.

LEE, Sang Ah. 2017. "The boundary-based view of spatial cognition: a synthesis." *Current Opinion in Behavioral Sciences* vol. 16: 58–65. DOI: https://doi.org/10.1016/j.cobeha.2017.03.006

LEIBNIZ, Gottfried Wilhelm. 1976. *Philosophical papers and letters.* 2nd ed., 2. print. (Synthese historical library vol 2). Dordrecht, Holland, Boston: D. Reidel Publishing Company.

LEVELT, Willem J. M. 1989. *Speaking: From intention to articulation*. *A Bradford book*. Cambridge, Mass., London: MIT Press.

LI, Peggy, & Lila GLEITMAN. (2002). "Turning the tables: language and spatial reasoning." *Cognition* vol. 83, issue 3: 265–294. DOI: https://doi.org/10.1016/S0010-0277(02)00009-4.

LIPTON, Jennifer S., & Elizabeth S. SPELKE. 2005. "Preschool children's mapping of number words to nonsymbolic numerosities." *Child Development* vol. 76, No. 5: 978–88. DOI: https://doi.org/10.1111/j.1467-8624.2005.00891.x.

MARZECOVÁ, Anna, Marcin BUKOWSKI, Ángel CORREA, Marianna BOROS, Juan LUPIÁÑEZ, & Zofia WODNIECKA. 2013. "Tracing the bilingual advantage in cognitive control: The role of flexibility in temporal preparation and category switching." *Journal of Cognitive Psychology* vol. 25, issue 5: 586–604. DOI: https://doi.org/10.1080/20445911.2013. 809348.

MCCRINK, Koleen, & Karen WYNN. 2004. "Large-number addition and subtraction by 9-month-old infants." *Psychological Science* vol. 15, issue 11: 776–81. DOI: https://doi.org/10.1111/j.0956-7976. 2004.00755.x

NĘCKA, Edward, Jarosław ORZECHOWSKI, & Błażej SZYMURA. 2006. *Psychologia poznawcza*. Warszawa: Wydawnictwo Naukowe PWN, Academica Wydawnictwo SWPS.

NIEDER, Andreas, & Stanislas DEHAENE. 2009. "Representation of number in the brain." *Annual Review of Neuroscience* vol. 32: 185–208. DOI: https://doi.org/10.1146/annurev.neuro.051508.135550.

PICA, Pierre, Cathy LEMER, Véronique IZARD, & Stanislas DEHAENE. 2004. "Exact and approximate arithmetic in an Amazonian indigene group." *Science* vol. 306 (5695): 499–503. DOI: https://doi.org/10.1126/science.1102085.

PICOZZI, Marta, Maria Dolores de HEVIA, Luisa GIRELLI, & Viola MACCHI CASSIA. 2010. "Seven-month-olds detect ordinal numerical relationships within temporal sequences." *Journal of Experimental Child Psychology* vol. 107, issue 3: 359–67. DOI: https://doi.org/10.1016/j.jecp.2010.05.005.

RUGANI, Rosa, Laura FONTANARI, Eleonora SIMONI, Lucia REGOLIN, & Giorgio VALLORTIGARA. 2009. "Arithmetic in newborn chicks." *Proceedings of the Royal Society B: Biological Sciences* vol. 276 (1666): 2451–2460. DOI: https://doi.org/10.1098/rspb.2009.0044.

SARNECKA, Barbara W., Valentina G. KAMENSKAYA, Yuko YAMANA, Tamiko OGURA & Yulia B. YUDOVINA. 2007. "From grammatical number to exact numbers: early meanings of 'one', 'two', and 'three' in English, Russian, and Japanese." *Cognitive Psychology* vpl. 55, issue 2: 136–68. DOI: https://doi.org/10.1016/j.cogpsych.2006.09.001.

SMITH, Edward E. 1999. “Working Memory.” In Robert A. WILSON & Frank KEIL (eds.). *The MIT encyclopedia of the cognitive sciences*, 888–889. Cambridge, Mass.: MIT Press.

SPELKE, Elizabeth S. 2003. “What makes us Smart: Core knowledge and natural language.” In Dedre GETNER & Susan GOLDIN-MEADOW (eds.). *A Bradford book. Language in mind: Advances in the study of language and thought*, 277–311. Cambridge, Mass., London: MIT Press.

SPELKE, Elizabeth S., & Sang Ah LEE. 2012. “Core systems of geometry in animal minds.” *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, vol. 367 (1603): 2784–93. DOI: https://doi.org/10.1098/rstb.2012.0210.

SPELKE, Elizabeth S., & Sanna TSIVKIN. 2001. “Language and number: a bilingual training study.” *Cognition* vol. 78, issue 1: 45–88. DOI: https://doi.org/10.1016/S0010-0277(00)00108-6.

TRICK, Lana M., & Zenon W. PYLYSHYN. 1994. “Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision.” *Psychological Review*, vol. 101, issue 1: 80–102. DOI: https://doi.org/10.1037/0033-295X.101.1.80.

VAN MARLE, Kristy, & Karen WYNN. 2011. “Tracking and quantifying objects and non-cohesive substances.” *Developmental Science* vol. 14, issue 3: 502–15. DOI: https://doi.org/10.1111/j.1467-7687.2010.00998.x.

WODNIECKA, Zofia, Karolina MIESZKOWSKA, Joanna DURLIK, & Ewa HAMAN. (2018). “Kiedy 1 + 1 ≠ 2, czyli jak dwujęzyczni przyswajają i przetwarzają język(i): When 1 + 1 ≠ 2: how bilinguals acquire and process language(s).” In Ewa CZAPLEWSKA (ed.). *Logopedia międzykulturowa*, 92–131. Gdańsk: Harmonia Universalis.

WYNN, Karen. 1992a. “Addition and subtraction by human infants.” *Nature* vol. 358 (6389), 749–750. DOI: https://doi.org/10.1038/358749a0.

WYNN, Karen. 1992b. “Children’s acquisition of the number words and the counting system.” *Cognitive Psychology* vol. 24, issue 2: 220–51. DOI: https://doi.org/10.1016/0010-0285(92)90008-P.

XU, Fei. 2003. “Numerosity discrimination in infants: Evidence for two systems of representations.” *Cognition* vol. 89, issue 1: B15-B25. DOI: https://doi.org/10.1016/S0010-0277(03)00050-7.

XU, Fei., & Susan CAREY. 1996. “Infants’ metaphysics: the case of numerical identity.” *Cognitive Psychology* vol. 30, issue 2: 111–53. DOI: https://doi.org/10.1006/cogp.1996.0005.

XU, Fei., & Elizabeth S. SPELKE. 2000. “Large number discrimination in 6-month-old infants.” *Cognition* vol. 74, issue: 1, B1-B11. DOI: https://doi.org/10.1016/S0010-0277(99)00066-9.

LANGUAGE AS A NECESSARY CONDITION FOR COMPLEX MENTAL CONTENT: A REVIEW OF THE DISCUSSION ON SPATIAL AND MATHEMATICAL THINKING

Summary

In this article we review the discussion over the thesis that language serves as an integrator of contents coming from different cognitive modules. After presenting the theoretical considerations, we examine two strands of empirical research that tested the hypothesis — spatial cognition and mathematical cognition. The idea shared by both of them is that each is composed of two separate modules processing information of a specific kind. For spatial thinking these are geometric information about the location of the object and the information about the object’s properties such as color or size. For mathematical thinking, they are the absolute representation of small numbers and the approximate representation of numerosities. Language is said to integrate the two kinds of information within each of these domains, which the reviewed data demonstrates. In the final part of the paper, we offer some comments on the theoretical side of the discussion.

JĘZYK JAKO WARUNEK KONIECZNY ZŁOŻONEJ TREŚCI MENTALNEJ: PRZEGLĄD BADAŃ NAD MYŚLENIEM PRZESTRZENNYM I MATEMATYCZNYM

Streszczenie

W niniejszym artykule dokonujemy przeglądu badań z zakresu psychologii poznawczej, które skupiają się na hipotezie języka jako integratora treści zaczerpniętych z oddzielnych modułów poznawczych. W pierwszej kolejności przedstawiamy teoretyczną stronę badań, a następnie przechodzimy do prezentacji dwóch obszarów badan empirycznych eksplorujących hipotezę języka jako integratora treści. Punktem wyjścia tych badań jest fakt, że w obydwu przypadkach mamy do czynienia z dwoma rodzajami informacji, przetwarzanych przez dwa oddzielne moduły. Dla myślenia przestrzennego są to informacja geometryczna na temat lokacji przedmiotu w przestrzeni oraz informacja na temat właściwości inherentnych przedmiotowi, takich jak kolor czy wielkość. W przypadku matematycznego myślenia, dwa moduły przetwarzają kolejno informację na temat absolutnych ale małych ilości oraz przybliżonych wielkości. Celem badań w tych dwóch obszarach jest wykazanie, że język jest koniecznym warunkiem ku temu, aby informacja z obydwu modułów została zintegrowana. W końcowej części artykułu oferujemy kilka komentarzy na temat teoretycznej strony przedstawionych badań.

Key words: mathematical thinking; modularity; spatial thinking; language.
Słowa kluczowe: myślenie matematyczne; modularność; myślenie przestrzenne; język.

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